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Subject:- Economics

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Unit No 1

Role of Mathematics in Economics

Mathematics is a science of space numbers. It is nomore a branch of economics. However, tools of mathematics are frequently applied to interpret economics phenomena. Thus mathematical analysis is merely an approach to economic analysis. It should not and does not differ from the non-mathematical analysis. The purpose of any theoretical analysis is to derive a set of conclusions or theorems from a given set of assumptions via a process of reasoning. In mathematical economics the assumptions and conclusions are stated in mathematical symbols rather than in words and in equations rather than in sentences.

The use of mathematics in economics substantiates the following advantage;,

(i) The language of mathematics used is more concise and precise.
(ii) There exists a rich treasure of mathematical theorems at our disposal which can be used to interpret economic theories.
(iii) It enables us to distinguish between different variables such as exogenous and endogenousvariablses, implicit and explicit variables involved in the economic theories.

Thus mathematical economics refers to application of mathematics to purely theoretical aspects of economic analysis with little or no error of measurement of the variables under study.

Some mathematical tools used in economics

1. **Matrices and Determinants:** Matrices and determinants are applied in various phenomena of economic analysis and management such as linear programming, game theory, General equilibrium analysis, input-output analysis etc.
2. **Functions, Limits and Continuity:**The concepts of functions, limits and continuity are of fundamental importance in economics. These concepts enable us to have a clear understanding of the nature of the function, and its derivatives i.e the study of changes that occur in one variable as a result of change in other (dependent) variable. These concepts are very useful in demand, supply analysis and production and consumption function.
3. **Maxima and Minima**: The concepts of maxima and minima play a very useful role in almostall fields of micro and macro economics. The concepts are used to understand utility maximization of the consumers, profit maximization and cost minimization of the producers.
4. **Differential equations and difference equations**: Differential equations are used frequently in economics. They are used to establish the conditions for dynamic stability micro economic

models of market equilibrium and to trace the time path of economic variables under various conditions. They are used to find total cost and total revenue functions from the marginal cost and marginal revenue functions.

Difference equations are widely used in economics to determine the conditions of dynamic stability in lagged economic models such as cobweb model, Harrod- Domer Growth model etc.

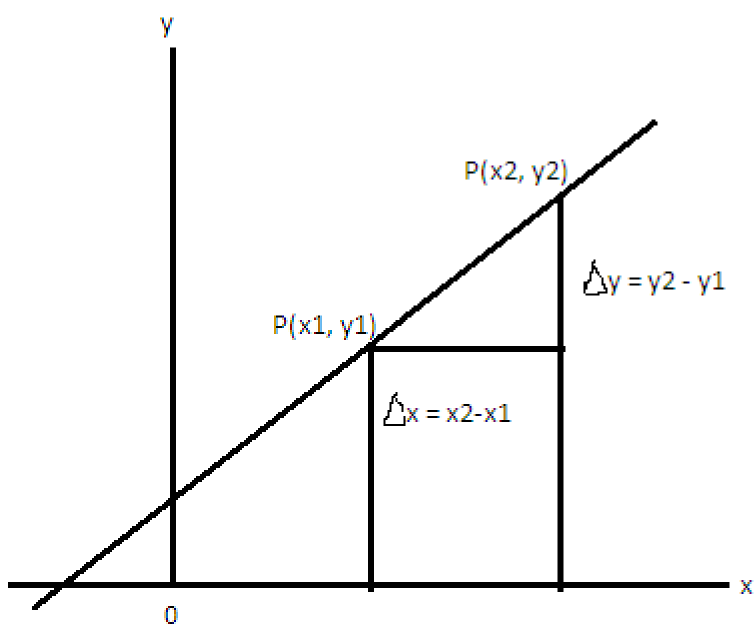
Coordinate Geometry

Straight Line: A straight line is defined as a connection of two collinear points. The equation of a straight line is generally written as

Y = mx + c

Which is a linear equation. Here 'm' is called the slop of the line and 'c' is called the intercept.

Slope of line: The slope of line is a measure of steepness. The slope of line measures the change in the dependent variable y (Δy) divided by a change in the independent variable x (Δx). The greater of absolute value of the slope, the steeper the line and vice versa. A positively sloped line moves up from left to right and a negatively sloped straight line moves down from left to right. The slope of a horizontal line for which Δy = 0, is zero. The slope of a vertical line for which Δx = 0 does not exist.



Let $P(x_1, y_2)$ and $P(X_2, Y_2)$ are two points on a line. Then the slope m of the line is defined as

$m = (\Delta y / \Delta x) = (y_2 - y_1) / (x_2 - x_1)$ = vertical change / horizontal change

Solved problems

Q 1. Find the distance between two points (1,4) and (3,5).

Ans. To find the distance between any two points. We use the formula

AB = $\sqrt{} (x_2 – x_1)^2 + (y_2 – y_1)^2$

Here x_1 = 1, x_2 = 3 and y_1 =4, y_2 = 5

Therefore AB = $\sqrt{}$ (3 -1)^2 + (5 -4)^2 = $\sqrt{}$4 +1 = $\sqrt{}$5.

Q2. Find the distance between two points (4, -6) and (3, 8).

Ans. Here x1 = 4, x2 = 3 and y1 = -6, y2 = 8

Therefore AB = $\sqrt{}(x_2 –x_1)^2 + (y_2 –y_1)^2$ = $\sqrt{}$ (3-4)^2 + (8+6)^2 = $\sqrt{}$ 1^2 + 14^2 = $\sqrt{}$ 1 + 196 = $\sqrt{}$ 197.

Q3. Find the slope of line passing through two points (-1,0) and (1,0).

Ans. The slope m = (Δy)/(Δx) = (y_2-y_1)/ (x_2-x_1)= 0-0/1- (-1) = 0/ 2 = 0.

Q4. Find the slope of the line passing through points (0.0) and (2,5)

Ans. Here x1 =0, x2 =2 and y2= 0, y1= 5

Slope (m) = (Δy)/(Δx) = (y_2-y_1)/ (x_2-x_1)= 5-0/2-0 = 5/2.

Q5. Find the slope of line 3y = 9x -2

Ans. 3y = 9x -2 can be re written as

Y = (9x -2)/ 3 = 3x – 2/3

Comparing with y = mx +c we get

Slope (m) = 3

Q6. Find the slope and intercept of the line($\sqrt{}$3x)+ y =12.

Ans. ($\sqrt{}$3x) +y =12 or y = - ($\sqrt{}$3x) + 12

Comparing with y = mx + c we get

Slope (m) = -$\sqrt{}$3, and intercept (c) = 12

Q7. Find the equation of a straight line whose slope is -2/5 and intercept is 6.

Ans.Given $m = -2/5$ and $c=6$

Therefore the equation of the st. line is $y = mx + c$

i.e $y = (-2/5)\,x + 6$ or $5y = -2x + 6\times 5$

or $5y + 2x - 30 = 0$ is the required equation.

Equation of a straight line in intercept form

The equation of a st. line making intercepts 'a' and 'b' on x-axis and y-axis is respectively is given by

$x/a + y/b = 1$

Q8. Find the equation of a st. line making intercepts 2 and -5 on x-axis and y-axis respectively

Ans. The equation of a st. line in intercept form is $x/a + y/b = 1$

Here $a = 2$, $b = -5$, therefore the required equation is $x/2 + y/-5 = 1$

Or $5x - 2y = 10$ (Ans)

Q9.Find equation of a st. line passing through points (3,5) and (4,7).

Ans. The equation of a st. line passing through two points (x_1,y_1) and (x_2,y_2) is given by

$$y - y_1 = [(y_2 - y_1)/(x_2 - x_1)]\,(x - x_1)$$

here $x_1 = 3$, $x_2 = 4$ and $y_1 = 5$, $y_2 = 7$

so the equation is $y - 5 = [(7-5)/(4-3)]\,(x-3)$

or $y - 5 = 2\,(x-3)$

or $y - 5 = 2x - 6$

hence $y = 2x - 1$ Ans.

Q10. Find the linear demand function for the following demand schedule for watches sold at 2 different prices

No. of watches sold	price per unit
20	90
30	60

Ans. Here demand curve passes through two points with co-ordinates (20,90) and(30,60)

The linear demand curve passing through 2 points (x_1,y_1) and (x_2,y_2) is given by

$y - y_1 = [(y_2-y_1)/(x_2-x_1)]\ (x-x_1)$

Where x = quantity demand and y = price

Therefore the linear demand curve is

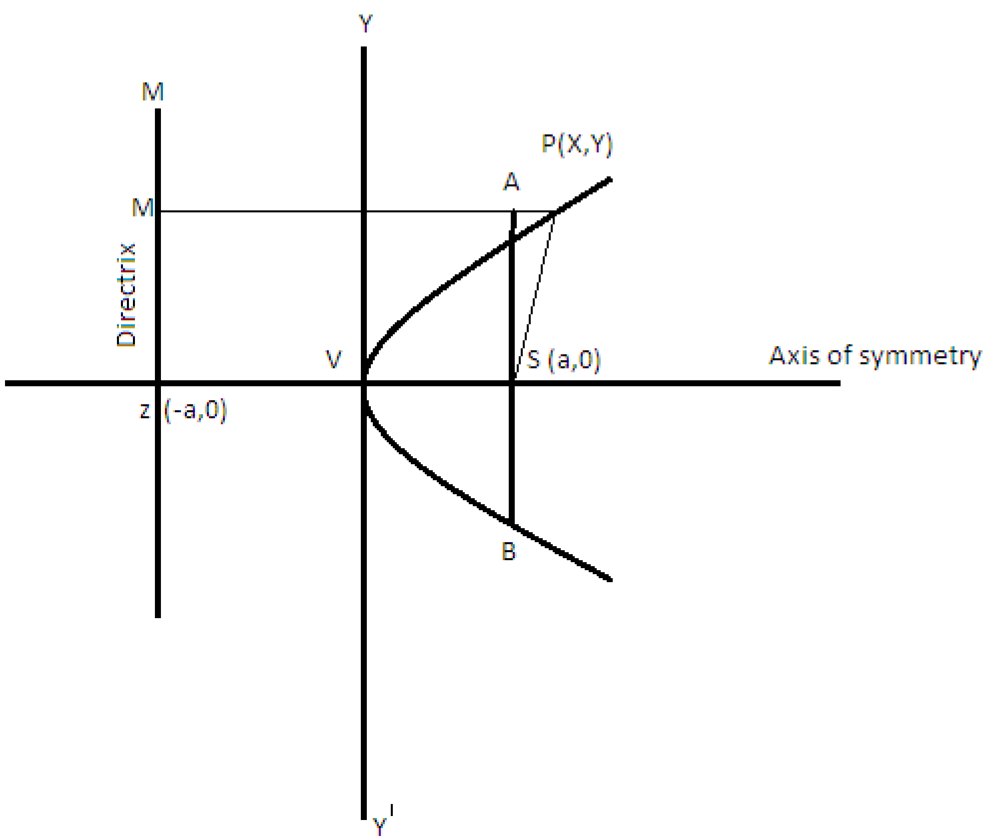
Y -90 = [(60-90)/(30-20)](x-20)

Or y-90 = -30/10 (x -20) = -3 (x -20) = -3x +60

Or y + 3x -150 = 0 Ans.

Parabola

Definition: A parabola is a set of points each of which is equidistant from a given point called the focus (s) and from a given line called a directrix (ZM). The directrix is parallel to and at a distance 'a' from the y-axis. Its equation is x +a =0

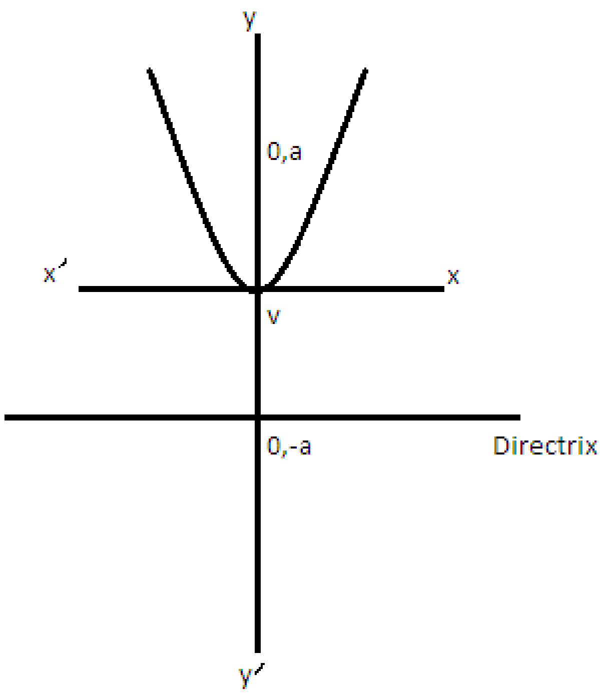


The line through the focus (s) and perpendicular to the directrix (ZM) is called the 'axis' of the parabola. The point on the axis midway between the focus (s) and directrix (ZM) is called the vertex. The line segment joining two points of a parabola and is known as Latus rectum whose length is equal to 4a.

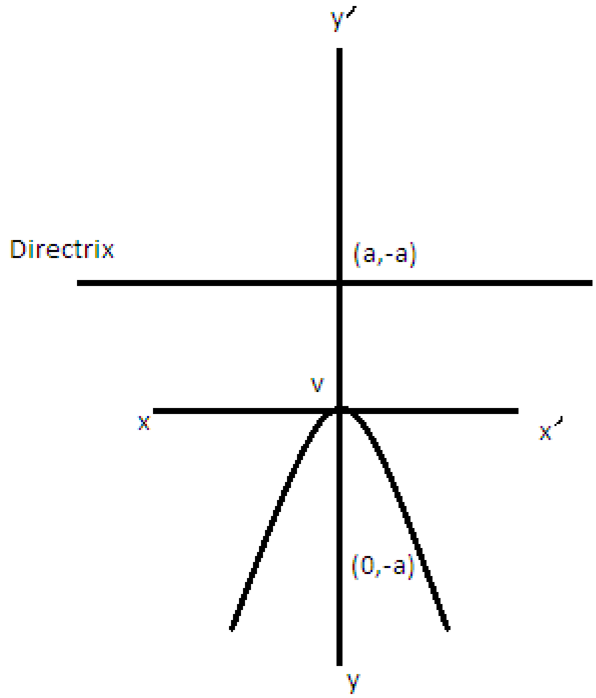
The equation of a parabola with vertex at origin is given by $y^2 = 4ax$. This is a right handed parabola.

The equation of a left handed parabola is similarly given by $y^2 = -4ax$.

The equation of an upward parabola is given by $y^2 = 4ay$.



Similarly the equation of a downward parabola is given by $x^2 = -4ay$



If the vertex of the parabola is the point (h,k) instead the point of origin, the above four cases become

Equation		Directix
(i)	$(y-k)^2 = 4a\ (x-h)$	$x = h – a$
(ii)	$(y-k)^2 =- 4a\ (x-h)$	$x = h +a$
(iii)	$(x-h)^2 = 4a\ (y-k)$	$y = k - a$
(iv)	$(x-h)^2 = 4a\ (y-k)$	$y = k +a$

Q1. Find the focus and directrix of the parabola $y^2 = 8x$.

Sol. $y^2 = 8x$.

Compare it with the general equation of the parabola $y^2 = 4ax$; we get

$4a = 8$ or $a = 2$

Therefore co-ordinates of focus are (a,0) = (2,0)

Directrixis $x + a = 0$ or $x + 2 = 0$

Q2. Find the focus ,directrix and lalusrectrum of the parabola. $y^2 = -16x$

Sol. $y^2 = -16x$

Compare it with left handed parabola $y^2 = -4ax$. We get

i.e $-4a = -16$ or $a = 4$

Therefore the co-ordinates of focus are $(-4,0)$

The directrix is $x - a = 0$ or $x - 4 = 0$

L. R = $4a = 4 . 4 = 16$.

Q3. Find the standard form for the parabola $y^2 + 2y - 12x - 3 = 0$

Sol. $y^2 + 2y - 12x - 3 = 0$

Or $y^2 + 2y + 1 = 12x + 4$

Or $(y + 1)^2 = 12 (x + 1/3)$

Compare it with $(y - k)^2 = 4a (x - h)$. we get

$K = -1, h = -1/3$

or vertex is $(-1/3, -1)$

$4a = 12$ or $a = 3$

Therefore the directrix is $x = h - a = (-1/3) - 3 = -10/3$

Or $x + 10/3 = 0$

The focus is $(h+a, k) = (-1/3)+3, -1 = 8/3, -1$

Q4. Find the vertex focus and directrix for the parabola $y = x^2 + 4x$

Sol. $y = x^2 + 4x$

$(x+4)^2 = y + 4$

Compare it with $(x - h)^2 = 4a (y - k)$ – case iii above we get

$h = -2, k = -4$

therefore vertex is $(h, k) = (-2,-4)$

since $4a = 1$ $a = ¼$

therefore focus is $(h, k+a) = (-2, -4+1/4) = (-2, -15/4)$

Directrix is $y = k - a$

Or $y = -4-1/4$ or $y = -17/4$

Or $y + 17/4 = 0$ Ans

Q5. Find the co-ordinates of vertex, focus, equation of directrix and latus rectrum for the parabola

$X^2 - 2x - 12y + 25 = 0$

Sol. $X^2 - 2x - 12y + 25 = 0$

$X^2 - 2x = 12y - 25$

$(x-1)^2 - 1 = 12y - 25$ or $(x-1)^2 = 12y - 24$

$(x-1)^2 = 12(y-2)$ which is an upword parabola.

Compare it with general form of upword parabola $(x-h)^2 = 4a(y-k)$ we get

$h = 1, k = 2$

therefore coordinates of vertex are $(h,k) = (1,2)$, since $4a = 12$ or $a = 3$

coordinates of focus are $(h, k+a) = (1, 2+3) = (1,5)$

Directrix is given by $y = k - a$

Or $y = 2-3 = -1$ or $y = -1$ or $y + 1 = 0$

$L.R = 4a = 4.3 = 12$

Q6. A firm produces an output at variable cost given by $\Pi = ax^3 - bx^2 + cx$. Show that average variable cost (AVC) is a parabola

Sol. $VC = ax^3 - bx^2 + cx$

Therefore $AVC = (ax^3 - bx^2 + cx)/x = ax^2 - bx + c$.

Denoting AVC by y. we get

$Y = ax^2 - bx + c$

$ax^2 - bx = y - c$

$a(x^2 - bx/a) = y - c$ or $a[(x - b/2a)^2 - b^2/4a^2] = y - c$

$a[(x - b/2a)^2] = y - c + b^2/4a^2 = y - (4ac - b^2)/4a$ or $(x - b/2a)^2] = 1/a[y - (4ac - b^2)/4a]$

which is of the form $(x - h)^2 = 4a(y - k)$ is an upword parabola

Q7. Show that the demand curve corresponding to the demand law $P = 4 – (x^2/100)$is a parabola

Sol.$P = 4 – (x^2/100)$ where P = price and x =demand

Therefore $TR = P.x = x (4- x^2/100) = 4x – x^3/100$

Therefore $A.R = T.R/ x =[4x – (x^3/100)]/x = 4 – x^2/100$

Since AR is also the demand curve and denoting AR by y we get

$Y =4 – x^2/100$ or $x^2/100 = 4 -y$ or $x^2 = -100(y-4)$

Which is a downword parabola of the form $(x –h)^2 = -4a (y –k)$. so the demand curve is a parabola.

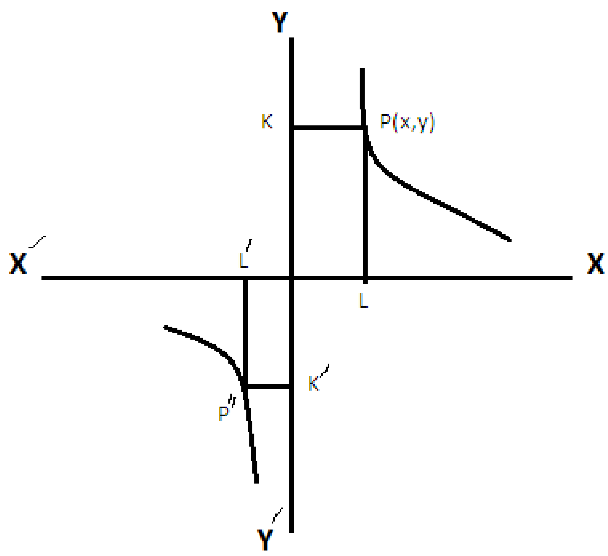
Rectangular Hyperbola

Definition: A rectangular hyperbola is defined as the locus of a point which moves in such a way that the product of its perpendicular distance from a fixed line to each is a positive constant say C^2. Fixed lines perpendicular to each other are called asymptotes and their point of intersection is called the centre of rectangular hyperbola.

Equation of rectangular Hyperbola

By definition PLx PK = c^2 where c^2 is a constant i.e$xy = c^2$ which is a equation of a rectangular hyperbola. In case the asymptotes of rectangular hyperbola are parallel to axis and the centre of rect. Hyp. is (a,b), then the equation of the rectangular hyperbola becomes

$(x –a) (x – b) = c^2$



Average fixed cost defined as total fixed cost divided by level of output i.e TFC/x is represented by a rectangular hyperbola. In this case output axis and cost axis are the asymptotes and the product of the distance of any point on AFC curve from the two axis is always equal to fixed cost (a positive constant)

Q. if the total cost curve is $\Pi = ax(x+b)/x + c$, where a, b and c are +ve constants. Show that average cost curve is rectangular hyperbola.

Sol. $TC = \Pi = ax(x+b)/x + c$

Therefore $AC = \Pi/x = [ax(x+b)/x + c]/x = a(x+b)/x + c$

Let AC is denoted by y we get

$Y = a(x+b)/x + c$ or $y(x+c) = a(x+b)$ or $xy + yc = ax + ab$

$ax - ay - cy = -ab$ or $ax + ac - xy - cy = -ab + ac$

$a(x+c) - y(x+c) = -ab + ac$

$(x+c)(a-y) = -ab + ac$

$(x+c)(y-a) = ab - ac$ which is the form $(x-a)(y-b) = c^2$, so it is a rectangular hyperbola.

Functions

Definition: A function f is a rule which assigns to each value of a variable (x) called the argument of the function, one and only one value [f(x)] referred to as the value of the function at x. The domain of a function refers to the set of all possible values of x; the range is the set of all possible values for f(x). functions are generally defined by algebraic formulas. A function is usually expressed as

Y = f(x) in which y is dependent variable and x is independent variable and 'f' denotes the unspecified relationship between y and x. It is a single valued function since there is a unique y in the range for each specified x. The converse may not necessarily be true.

Types of function

Functions frequently used in economics are

1.**Constant functions:** A zero degree polynomial function is a called a constant function e.g f(x) = k where k = constant.

2. **Linear Function:** A polynomial function of degree 1 is called a linear function e.g f(x) = mx + c. where m and c are constants.

3. **Quadratic Function:** A polynomial function of degree 2 is called aQuadratic function e.g

$f(x) = ax^2 + bx + c$ where $a \neq 0$

4.**Polynomial function:** A function of the form

$f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots\dots.. + a_o$ where n is non- negative integer, $a_n \neq 0$ is called apolynomial function

5.**Rational function:** A function expressed as the ratio of two polynomials is called a rational function. Thus y = f(x)/g(x) where f(x) and g(x) are both polynomials in x is a rational function.

6.**Even function:** A function f is said to be an even function if f (-x) = f(x). e.g $f(x) = x^2 + 4$ is an even function for all values of x.

7.**odd function**: A function f is said to be odd function if f(-x) = -f(x) e.g. $f(x) = x^3$ is an odd function for all values of x.

8.**Monotone Function:** A function is monotone if it is either increasing or decreasing. A function is increasing if $f(x_1) < f(x_2)$ for $x_1 < x_2$.

A function is decreasing if $f(x_1) > f(x_2)$ for $x_1 < x_2$.

9.**Non Algebraic function:** A function is said to be non algebraic function if the relation which involves the finite terms and variables are not affected by the operation of addition, subtraction, multiplication, division, powers and roots, exponential function, logarithmic function, trigonometric function are called non-algebraic functions.

Solved Problems

Q1.Given $f(x) = x^2 + 4x - 5$. Find f(2) and f (-3)

Sol. (i) $f(x) = x^2 + 4x - 5$

Put x = 2 we get $f(2) = 2^2 + 4.2 - 5 = 4 + 8 - 5 = 7$

(ii) Put x = -3 we get

$F(-3) = (-3)^2 + 4.(-3) - 5 = 9 – 12 – 5 = -8$ Ans

Q2. $f(x) = x^2 - 5x + 3$ (i) f(0), (ii) f (-2) (iii) f(1/2)

Sol. (i) $f(x) = x^2 - 5x + 3$

Put x = 0 we get f(0) = 0 – 5.0 +3 = Ans.

(ii) put x = -2 we get

$f(-2) = (-2)^2 - 5(-2) + 3\ 4 + 10 + 3 = 17$ Ans

(III) put x = 1/2 we get

$f(1/2) = (1/2)^2 - 5(1/2) + 3 = ¼ - 5/2 + 3 = (1 – 10 + 12)/4 = ¾$ Ans.

Q3. If $f(x) = e^{-x}$, find f(-a)/f(b)

Sol. $F(x) = e^{-x}$

Put x = -a we have $f(-a) = e^{-(-a)} = e^a$

Put x = b we get $f(b) = e^{-b}$

Therefore $f(-a)/f(b) = e^a / e^{-b} = e^{a-(-b)} = e^{a+b}$Ans

Q4. If $f(x) = x^2 - 5x + 3$ then find f(f(x))

Sol. $f(x) = x^2 - 5x + 3$

Put $x = f(x) = x^2 - 5x + 3$

Therefore $f(f(x)) = (x^2 - 5x + 3)^2 - 5(x^2 - 5x + 3) + 3 = x^4 + 25x^2 + 9 - 10x^3 + -30x + 6x^2 - 5x^2 + 25x - 15 + 3$

$f(f(x)) = x^4 - 10x^3 + 26x^2 - 5x - 3$ Ans

Q5. If f(x) = (1/x) +ax and f(1/5) = 28/5 find the value of a.

Sol. f(x) = (1/x) + ax

Put x = 1/5 we get f(1/5) = (1/1/5)+ a.(1/5)

f(1/5) = 5 + a/5 since f(1/5) = 28/5 we get

5 +a/5 = 28/5   or a/5 = (28/5)- 5 = 3/5   or a = 3 Ans

Limits

Definition: If the functional values f(x) of a function f draw closer to one and only one finite real number L for all values of x as x draws closer to 'a' from both sides, but does not equal a, L is defined as the limit of f(x) as x approaches 'a' and is written as

$$\lim_{x\to a} f(x) = L$$

Solved problems

Q1. Evaluate (i) $\lim_{x\to 7} (x-7)/x^2-49$ (ii)) $\lim_{x\to -7} (x-7)/x^2-49$ (iii)) $\lim_{x\to 2} (x^2-4)/x-2$ (iv) $\lim_{x\to -3} (x^2-9)/x-3$

Sol. (i) $\lim_{x\to 7} (x-7)/x^2-49 = \lim_{x\to 7} (x-7)/(x-7)(x+7) = 1/(7+7) = 1/14$ Ans

(ii)) $\lim_{x\to -7} (x-7)/x^2-49 = \lim_{x\to -7} (x-7)/(x-7)(x+7) = 1/(17+7) = 1/0$

Hence limit does not exist.

(III) $\lim_{x\to 2} (x^2-4)/x-2$ By factorizating the numerator

$$\lim_{x\to 2} (x+2)(x-2)/(x-2) = -2+2 = 0 \text{ Ans}$$

(iv) $\lim_{x\to -3} (x^2-9)/x-3$ By factorizating the numerator

$$\lim_{x\to -3} (x-3)(x+3)/(x-3) = -3+3 = 0 \text{ Ans}$$

Q2. Evaluate $\lim_{x\to 6} (x^2 - 2x - 24)/x-6$

Sol By factorizing the numerator

$\lim_{x\to 6} (x-6)(x+4)/x-6 = \lim_{x\to 6} (x+4) = 6+4 = 10$ Ans

Q3. Evaluate $\lim_{x\to\infty} (3x^2 - 7x)/4x^2-21$

Sol. As $x\to\infty$ numerator and denominator become infinite. In such a case we must divide all terms by the highest power of x in the function.

Thus divide both numerator and denominator by x^2

$\lim_{x\to\infty} (3x^2 - 7x)/ 4x^2 - 21 = \lim_{x\to\infty} [\{(3x^2 - 7x)/x^2\}/ (4x^2 - 21)/x^2] = \lim_{x\to\infty} (3 - 7/x)/ (4 - 21/x^2) = ¾$

since as $x\to\infty$, $1/x$, $1/x^2$, $1/x^3 \to\infty$

Q4. Evaluate $\lim_{x\to\infty} 4x^3 - 7x^2 + 8x)/ 4x^4 + 8x^2$

Do your self

Q5 Prove that $\lim_{h\to 0} [(x+h)^m - x^m]/h = mx^{m-1}$

Sol. $\lim_{h\to 0} [(x+h)^m - x^m]/h = \lim_{h\to 0} [x^m (1 + h/x)^m - x^m]/h$

Expanding by bi-nominal theorem

$= \lim_{h\to 0} [x^m (1 + m\, h/x) + m(m-1)/2!\, (h/x)^2 + m(m-1)(m-2)/3!\, (h/x)^3 + \ldots\ldots\ldots\ldots\ldots - x^m]/h$

$= \lim_{h\to 0} [x^m (1 + m\, h/x) + m(m-1)/2\, (h/x)^2 + m(m-1)(m-2)/6\, (h/x)^3 + \ldots\ldots\ldots\ldots\ldots - 1]/h$

$= \lim_{h\to 0} [x^m (h/x) (m + m(m-1)/2\, (h/x) + m(m-1)(m-2)/6\, (h/x)^2 + \ldots\ldots\ldots\ldots\ldots]/h$

$= (x^m/x)\, m = x^{m-1}.\, m$Ans

Q6. Evaluate $\text{Limit}_{x\to 4} (x^2 - 16)/ (\sqrt{x^2 + 9}) - 5$

sol. $\text{Limit}_{x\to 4} (x - 16)/ (\sqrt{x + 9}) - 5$

it assumes the form 0/0 when x =4. By rationalization. We get

. $\text{Limit}_{x\to 4} (x^2 - 16)/ (\sqrt{x^2 + 9}) - 5 x [(\sqrt{x^2 + 9}) - 5]/[(\sqrt{x^2 + 9}) - 5]$

$=. \text{Limit}_{x\to 4} [(x^2 - 16) (\sqrt{x^2 + 9}) + 5]/ (\sqrt{x^2 + 9})^2 - 5^2$

$=. \text{Limit}_{x\to 4} [(x^2 - 16) (\sqrt{x + 9}) + 5]/ x^2 + 9 - 25$

$=. \text{Limit}_{x\to 4} [(x^2 - 16) (\sqrt{x^2 + 9}) + 5]/ (x^2 - 16)$

$=. \text{Limit}_{x\to 4} (\sqrt{x^2 + 9}) + 5 = (\sqrt{4^2 + 9}) + 5 = 5+5 = 10$ Ans

Continuity

Definition: A function which has no breaks in its curve is called a continuous function. It can be drawn with out lifting the pencil from the paper. A function 'f' is continuous at x = a, if

(1) F(x) is defined i.e exists at x =a
(2) $\lim_{x\to a} f(x)$ exists and
(3) $\lim_{x\to a} f(x) = f(a)$

All these three conditions must be satisfied for a function to be continuous

Q1. Indicate whether the following functions are continuous at the specified points

(i) $f(x) = 5x^2 - 8x + 9$ at $x = 3$

sol. $f(x) = 5x^2 - 8x + 9$ Put $x = 3$ we get

$f(3) = 5.3^2 - 8.3 + 9 = 45 - 24 + 9 = 30$

therefore f(3) exists

(ii) $\lim_{x \to 3} f(x) = \lim_{x \to 3} 5x - 8x + 9 = 5.3^2 - 8.3 + 9 = 30$

thereforelimit exists at x =3

since $f(3) = \lim_{x \to 3} f(x) = 30$

Hence f(x) is continuous at x =3

Q2. $f(x) = (x^2 + 3x + 12)/x-3$ at $x = 4$

Sol. $f(x) = (x^2 + 3x + 12)/x-3$ Put $x = 4$

(i) $f(4) = (4^2 + 3.4 + 12)/4-3 = 40/1 = 40$

(ii) $\lim_{x \to 4} (x^2 + 3x + 12)/x-3 = (4^2 + 3.4 + 12)/4-3 = 40/1 = 40$

since $f(4) = \lim_{x \to 4} f(x)$, the function is continuous at x =4

Q3. $f(x) = (x-3)/x^2-9$ at $x = 3$

Sol. $f(x) = (x-3)/x^2-9$

(i) $f(3) = (3-3)/3^2 - 9 = 0/0$

so f(3) does not exist

Thus the function is not continuous at x=3, even if its limit exists at x =3

(ii) $\lim_{x \to 3} (x-3)/x^2 - 9 = \lim_{x \to 3} x-3/(x-3)(x+3) = \lim_{x \to 3} 1/x+3 = 1/3+3 = 1/6$

(iii) Since $\lim_{x \to 3} f(x) = 1/6 \neq f(3)$

hence it is not continuous at x =3

Derivative

Definition: Given a function y = f(x), the derivative of the function written as f' (x) or dy/dx is defined as

$f'(x) = dy/dx = \lim_{\Delta x \to 0} [f(x + \Delta x) - f(x)]/\Delta x$

Or $f'(x) = dy/dx = \lim_{h \to 0} [f(x + h) - f(x)]/h$

Rules of differentiation

1. **Constant function Rule**: The derivative of a constant function f(x) =k, where k is constant is zero Example, given f(x) =8, f'(x) =0.
2. **Linear function Rule**: The derivative of linear function f(x) = mx +b is equal to m, the coefficient of x. f(x) = 3x +2. Then f'(x) = 3 Ans
   Given f(x)= 5 –(1/4)x, then f'(x) = - 1/4
3. **Power function Rule**: The derivative of a power function f(x) =kx^n where k is a constant and n is any real number is equal to the coefficient k times the exponent n multiplied by the variable x raised to power n-1.
   i.ef(x) = k. x^nthen f'(x) = km.x^{n-1}
   example f(x) = $4x^3$ then f'(x) = 4.3 x^{3-1} = 12 x^2Ans

4. **The Rule of Sums and Differences**: The derivative of a sum of two functions f(x) = g(x) + h(x) where g(x) and h(x) are both differentiable is equal to the sum of the derivative of the individual functions. Similarly, the derivative of the difference of two functions is equal to the difference of the derivatives of the function.
   Given f(x) = g(x) ±h(x), then f'(x) = g'(x) ±h'(x)
   Example (i) f(x) = 12 x^5 -$4x^4$ then f'(x) = 12.5x^4 – 4.4 x^3 = $60x^4$ -$16x^3$
(iii) f(x) = $9x^2$ +2x -3 then f'(x) = 9.2x +2 -0 = 18x +2 Ans
5. **The Product Rule**: The derivative of a product f(x) =g(x). h(x) where g(x) and h(x) are both differentiable functions, is equal to the first function multiplied by the derivative of the 2nd function plus the 2nd function multiplied by the derivative of the first function i.e
   Given f(x) = g(x) .h(x) then f'(x) =g(x).h'(x) +h(x).g'(x)
   Example Given f(x) = (3x)(2x-5) then dy/dx = f'(x) = $3x^3$ (d/dx)(2x-5) +(2x-5) d/dx($3x^3$)
   =$3x^3$. 2 +(2x-5)($9x^2$) = $6x^3$ + $18x^3$ -45 x^2 = 24 x^3 – 45 x^2Ans
6. **The quotient Rule**: The derivative of a quotient f(x) = g(x) / h(x) where g(x) and h(x) are both differentiable and h(x) ≠ 0 is equal to the denominator times the derivative of the numerator minus the numerator times the derivative of the denominator all divided by the denominator squared. Give f(x) = g(x) / h(x) then f'(x) = [h(x).g'(x) –g(x)h'(x)]/$[h(x)]^2$
   Example f(x) = $5x^3$/(4x+3) then f'(x) = [(4x+3)($15x^2$) -$5x^3$ (4)]/$(4x+3)^2$ = (60 x^3 +$45x^2$ -$20x^3$)/$(4x +3)^2$ = ($40x^3$ +$45x^2$)/ $(4x +3)^2$Ans

Solved Problems

1. Differentiate the following using appropriate rules (i) f(x) =17 (ii) y =-12 (iii) y =$8x^4$ (iv) y = -$6x^5$ (v)f(x) = 18 √x

Sol. (i) f(x) =17 therefore f'(x) = 0
(ii) Y = -12 therefore dy/dx = y'= 0
(iii) Y = $8x^4$, dy/dx = y' = 32 x^{4-1} = $32x^3$
(iv) Y = -$6x^5$, dy/dx = y' = -6.5 x^{5-1} = -30 x^4
(v) f(x) = 18 √x = $18x^{1/2}$, f'(x) = 18. ½ ($x^{(1/2)-1}$) = $9x^{-1/2}$ =9/√x Ans

Q2. Differentiate (i) y= ($4x^2$ -3) ($2x^5$) (ii) y = $7x^9$ ($3x^2$ – 12)
Sol. (i) y= ($4x^2$ -3) ($2x^5$) using product rule we have
dy/dx = ($4x^2$ -3) d/dx($2x^5$)+($2x^5$) d/dx ($4x^2$ -3) =($4x^2$ -3) ($10x^4$) + (8x) ($2x^5$)
= 40 x^6 -$30x^4$ +$16x^6$= $56x^6$ -$30x^4$
(ii) y = $7x^9$ ($3x^2$ – 12) using product rule

dy/dx =$7x^9$d/dx ($3x^2$ – 12) + ($3x^2$ – 12) d/dx ($7x^9$) =$7x^9$ (6x) +($3x^2$ – 12)$63x^8$ =$42x^{10}$ + $189x^{10}$ -$756x^8$

$=231x^{10} - 756x^8$ Ans

Q3. Differentiate (i) $y = (10x^8 - 6x^7)/2x$ (ii) $y = 4x^5 / 1-3x$ $x \neq 1/3$

Sol. (i) $y = (10x^8 - 6x^7)/2x$ then $dy/dx = [2x\, d/dx(10x^8 - 6x^7) - (10x^8 - 6x^7) d/dx(2x)]/(2x)^2$

$= [(80x^7 - 42x^6)2x - (10x^8 - 6x^7)2]/ 4x^2 = [(160x^8 - 84x^7) - (20x^8 - 12x^7)]/ 4x^2 = (140x^8 - 72x^7)/ 4x^2$

$= 4x^2(35x^6 - 18x^5)/ 4x^2 = (35x^6 - 18x^5)$ Ans

(ii) $y = 4x^5 / 1-3x$

$dy/dx = [d/dx(4x^5)(1-3x) - 4x^5\, d/dx(1-3x)]/ (1-3x)^2 = [(20x^4)(1-3x) - 4x^5(-3)]/ (1-3x)^2$

$=(20x^4 - 60x^5 + 12x^5)/ (1-3x)^2 = (20x^4 - 48x^5)/ (1-3x)^2$ Ans

Q3. If $y = (1 - \sqrt{x})/ 1+\sqrt{x})$ find out the dy/dx

Sol: $y = (1 - \sqrt{x})/ (1+\sqrt{x})$

Differential using quotient rule

$dy/dx = [(1+\sqrt{x})\ d/dx\ (1 - \sqrt{x}) - (1 - \sqrt{x})\ d/dx\ (1+\sqrt{x})]/(1+\sqrt{x})^2$

$= [(1+\sqrt{x})(-1/2x^{-1/2}) - (1 - \sqrt{x})(1/2x^{-1/2})]/(1+\sqrt{x})^2 = [(-1/2\sqrt{x})(1+\sqrt{x}) - (1/2\sqrt{x})(1-\sqrt{x})]/(1+\sqrt{x})^2$

$= (-1/2\sqrt{x}) -1/2 - 1/2\sqrt{x} + 1/2]/(1+\sqrt{x})^2 = -2/2\sqrt{x}]/(1+\sqrt{x})^2$

$dy/dx = -1/\sqrt{x}(1+\sqrt{x})^2$ Ans

Q4. A demand function is given as $q = 50 – 5p$. Compute the price elasticity of demand at $p=5$.

Sol Given $q = 50 – 5p$

Differentiate with respect to p we get.

$dq/dp = -5$, Now at $p = 5$, $q = 50 – 5.5 = 25$

$E_p = (-dq/dp)(p/q) = -(-5) . 5/25 = 1$ Ans

Q5. For the law of demand $q = 20/p+1$, find elasticity at $p=4$

Sol. Given $q = 20/ p+1$

Diff. w. r .t p we get $dq/dp = [(p+1)\ d/dp(20) - 20\ d/dp(P+1)]/(p+1)^2 = (p+1)(0) – 20 (1)]/(p+1)^2$

$dq/dp = -20/(p+1)^2$

now at p =4, dq/dp = $-20/(4+1)^2 = -4/5$

also at p =4, q = 20/4 +1 = 4

therefore E_p = -dq/dp (p/q) = -(-4/5) (4/4)= 4/5 Ans

Q6. Given the demand curve represented by p =100 – 5q (i) Find marginal revenue for any output (ii) Find MR when output (q=0) and q=4

Sol. P = 100- 5q, therefore R = p.q = q(100-5q) = $100q - 5q^2$

Diff, w r t q we get, dR/dq = 100 -10q, sincedR/dq = MR

Therefore MR = 100 – 10q

(ii) when q=0, MR =100 and when q =4 , MR = 100- 10(4) = 60 Ans

Q7. The total cost is given as C= $0.005x^3 - 0.02x^2 - 30x + 3000$Find total cost (i) when output (x) = 4

(ii) AC when output =10 (iii) MC when output =3

Sol. (i) C = $0.005x^3 - 0.02x^2 - 30x + 3000$ at x =4

C =$0.005(4)^3 - 0.02(4)^2 - 30(4) + 3000$ = 0.320 – 0.32 – 120 +3000 = 2880 Ans

AC = TC/x= C/x = $[0.005x^3 - 0.02x^2 - 30x + 3000]/x$

AC = $0.005x^2 - 0.02x - 30 + 3000/x$ at x =10

AC = $0.005(10)^2 - 0.02(10) - 30 + 300$ = 500 – 0.20 – 30 +300 = 270.3

MC = dc/dx = d/dx $(0.005x^3 - 0.02x^2 - 30x + 3000) = 3(0.005x^2) - 2(0.02x) - 30 = 0.015x^2 - 0.04x - 30$

At x =3 , MC = $0.015(3)^2 - 0.04(3) - 30$ =0.135 – 0.12 – 30 = - 29. 985 Ans

Integration: Differentiation is the process of finding the derivative f'(x) of a function f(x). Frequently in economics, however, we know that the rate of change of a function f'(x) and want to find the original function f(x). Reversing the process of differentiation and finding the original function from the derivative is called Integration. The original function f(x) is called the integral of f'(x)

Thus $\int f'(x) = f(x) + c$

Rules of Integration:

1. The integral of a constant is $\int k dx = kx + c$ where c is constant
2. $\int x^n dx = (x^{n+1}) / (n+1)$
3. $\int x^{-1} dx = \int 1/x \, dx = \log x + c$.
4. The integral of an exponential function is $\int a^{kx} dx = a^{kx}/(k \log a) + c$
5. The integral of a natutal exponential function is $\int e^{kx} dx = (e^{kx}/k) + c$
6. The integral of the sum or difference of two or more functions equals the sum or difference of their integrals $\int [f(x) + g(x)] dx = \int f(x) dx + \int g(x) dx$

Solved Problems

Q1 Evaluate (i) $\int x^4 dx$ (ii) $\int \sqrt{x} dx$ (iii) $\int 6x^2 dx$

Sol. (i) $\int x^4 dx = (x^{4+1}/4+1) + c = (x^5/5) + c$ Ans

(ii) $\int \sqrt{x} dx = \int x^{1/2} dx = (x^{1/2+1}/ ½ +1) + c = (x^{3/2}/3/2) + c = 2/3 \, x^{3/2} + c$ Ans

(iii) $\int 6x^2 dx = 6\int x^2 dx = 6 (x^{2+1} / 2+1) + c = 6/3(x^3) + c = 2 x^3 + c$ Ans

Q2. Evaluate $\int (3x^2 - x - 1) dx$

Sol.$\int (3x^2 - x - 1)dx = \int 3x^2 dx - \int x \, dx - \int dx = 3 (x^{2+1}/2+1) - x^{1+1}/1+1) - x + c = 3/3 (x^3) - 1/2 (x^2) - x + c$

$= x^3 - 1/2(x^2) - x + c$ Ans

Q3. Find $\int 3x^{-1} dx$

Sol. $\int 3x^{-1} dx = 3\int (1/)x \, dx = 3 \log x + c$ Ans

Q4. Find (i) $\int 9e^x dx$ (ii)$\int 9e^{-3x} dx$

Sol. (I) $\int 9e^x dx = 9\int e^x dx = 9e^x + c$, Since $e^x dx = e^x$

(ii)$\int 9e^{-3x} dx = 9\int e^{-3x} dx = 9 e^{-3x}/-3 + c = -3e^{-3x} + c$ Ans

Q5. Evaluate $\int x (x^2+1)^{3/2} dx$ by substitution method

Sol. Put $u = x^2 + 1$.

Diff. w r t x we get $du/dx = 2x$

Therefore $x \, dx = ½ \, du$

Hence $\int x (x^2 +1)^{3/2} dx = \int u^{3/2} (1/2) du = 1/2 \int u^{3/2} du = [½ (u^{(3/2)+1})/(3/2)+1] + c = [½ (u^{5/2})/5/2] + c$

$= [½ (u^{5/2})(2/5)] + c = 1/5 (u^{5/2}) + c$

Sub. The value of u we get

$1/5 (x^2 +1)^{5/2} + c$ Ans

Q6. Evaluate $\int [x/\sqrt{1+x^2}]dx$

Sol. $\int [x/\sqrt{1+x^2}]dx$ Put $1+x^2 = u$

Diff. w r t x we get

$du/dx = 2x$ therefore $xdx = (1/2) du$

$\int[x/\sqrt{1+x^2}]dx = 1/2\int[du/\sqrt{u}] = 1/2\int u^{-1/2}du = [½ (u^{(-1/2)+1})/(-1/2)+1] +c =1/2 [u^{1/2}/1/2] +c$

$=u^{1/2} +c$

Substitute the value of u we get $\sqrt{(1+x^2)} +c$ Ans

Q7. Evaluate $\int[x^5/x^{12}+1]dx$

Sol. Put $x^6 = u$ and diff w r t x we get

$du/dx =6x^5$ therefore $x^5\, dx = 1/6\, du$

$\int[x^5/x^{12}+1]dx = 1/6 \int[du/u^2+1] =1/6 \tan^{-1} u +c= 1/6 \tan^{-1} x^5 +c$ Because $\int dx/x^2+1 = \tan^{-1} x$

Q8. Evaluate $\int[2x -7/3 (x^2 -7x+6)^2]dx$

Put $x^2 -7x +6 =u$ and diff w r t x we get $du/dx = 2x -7$

$\int[2x -7/3 (x^2 -7x+6)^2]dx = \int du/3u^2 = 1/3\int (1/u^2) du = 1/3 \int u^{-2} du = 1/3 (u^{-2+1} /-2+1) +c = -1/3 u^{-1} +c$

$= -1/3 (1/u) +c =[-1/3 (1/ x^2 -7x +6)] +c$ ans

Q9 Evaluate$\int[e^x /1 +e^x]dx$

Sol. $\int[e^x /1 +e^x]dx$

Put $1 +e^x=u$ and diff. w r t x we get $du/dx = e^x$ and $du = e^x\, dx$

Therefore $\int[e^x /1 +e^x]dx = \int[du /u] = \log u +c$

Substitute the value of u we get

$\int[e^x /1 +e^x]dx = \log (1+e^x) +c$

Q10. If $MR = 800 -8q -18q^2$. Find TR and demand function.

Sol$MR = 800 -8q -18q^2$

$TR = \int MRdq +c =\int(800 -8q -18q^2)dq+c = \int 800dq -8\int qdq -18\int q^2dq +c$

$= 800q – (8q^2/2) – (18q^3/3) +c =800q – 4q^2 – 6q^3 +c$

Demand function P =Tr/q

Or $P =(800q – 4q^2 – 6q^3)/q = 800 – 4q – 6q^2$Ans

Q11. If marginal cost function is $MC = 1 + 2x +x^2$, Find total cost and cost function at fixed cost of Rs 100.

Sol. $MC = 1 + 2x +x^2$

Therefore TC =$\int$MC dx =$\int(1 + 2x + x^2)$ =$\int 1dx + \int 2x\,dx + \int x^2\,dx$ =$x + 2x^2/2 + x^3/3 + c$ =$x + x^2 + x^3/3 + c$

When x =0, fixed cost = 100 we have 100 = 0+ 0+ 0 +c or c = 100

Therefore TC = $x + x^2 + (x^3/3) + 100$

AC = TC/x =$[x + x^2 + (x^3/3) + 100]/x$ =$1 + x + (x^2/3) + 100/x$ Ans

Matrices

Definition of matrix: A matrix is a rectangular array of numbers, parameters or variables each of which has carefully ordered place within the matrix. The numbers are referred to as elements of the matrix. The numbers in the horizontal line are called rows; the numbers in the vertical line are called columns. The number of rows 'm' and columns 'n' defines the dimensions of the matrix (m×n)

Example

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times 3} \qquad B = \begin{bmatrix} 3 & 9 & 8 \\ 4 & 2 & 7 \end{bmatrix}_{2\times 3} \qquad C = \begin{bmatrix} 7 \\ 4 \\ 5 \end{bmatrix}_{3\times 1} = \text{Column Vector}$$

$$D = \begin{bmatrix} 3 & 0 & 1 \end{bmatrix}_{1\times 3} = \text{Row vector}$$

Types of matrix

1. **Zero or Null Matrix**: A matrix with every element zero is called a null or zero matrix. It is denoted by 0. It plays the role of zero in matrix theory.

$$0_{2\times 3} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ is a 2×3 null matrix.}$$

2. **Diagonal Matrix:** A square matrix in which all elements are zero except the elements of the leading diagonal is called a diagonal matrix denoted as D.

$$D = \begin{bmatrix} a_1 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{bmatrix} \text{ is a 3 x 3 diagonal matrix.}$$

3. **Upper and lower triangular matrix:** A square matrix all of whose elements below the main diagonal are zero is called upper triangular. If all elements above the main diagonal are zero it is lower triangular. For example

$\begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{bmatrix}$ is an upper triangular matrix.

$\begin{bmatrix} a & 0 & 0 \\ b & c & 0 \\ d & e & f \end{bmatrix}$ is an lower triangular matrix

4. **Square Matrix**: A square matrix is one in which the number of rows (m) is equal to the number of columns (n).

A = $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times3}$ is a square matrix.

5. **Identity matrix**: If in the diagonal matrix D each diagonal element is 1, it is called an identity matrix for example.

I = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is an identity matrix of order 3. It plays the role of the number 1 in matrices theory

6. **Symmetric matrix**: A square matrix such that its transpose (A') =A is called a symmetric matrix. Example

A= $\begin{bmatrix} 5 & a \\ a & 5 \end{bmatrix}$ by changing rows in columns we get its transpose A'

Therefore A'= $\begin{bmatrix} 5 & a \\ a & 5 \end{bmatrix}$ =A Hence a is a symmetric matrix.

7. **Singular and non-singular matrix**: A square matrix A is singular if |A| = 0 It is non- singular

if |A| ≠ 0 . Example

A= $\begin{bmatrix} 12 & 3 \\ 20 & 5 \end{bmatrix}$ Here |A| = $\begin{bmatrix} 12 & 3 \\ 20 & 5 \end{bmatrix} = 60 - 60 = 0$ Hence A is a singular matrix

Determinant

Evaluate the determinants

Q1. $\begin{vmatrix} 5 & 7 \\ 2 & 4 \end{vmatrix} = 5 \times 4 - 7 \times 2 = 20 - 14 = 6 Ans$

Q2. $\begin{vmatrix} 2 & -4 \\ 1 & 0 \end{vmatrix} = 2 \times 0 - (-4) \times 1 = 0 + 4 = 4 Ans$

Q3. $\begin{vmatrix} m_1 & b_1 \\ m_2 & b_2 \end{vmatrix} = m_1 b_2 - b_1 m_2 Ans$

Q4. $\begin{vmatrix} 9 & 6 & 4 \\ 3 & 2 & 1 \\ 4 & 1 & 5 \end{vmatrix}$ = 9 $\begin{vmatrix} 2 & 1 \\ 1 & 5 \end{vmatrix}$ - 6 $\begin{vmatrix} 3 & 1 \\ 4 & 5 \end{vmatrix}$ +4 $\begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$ =9(2×5-1×1) -6(3x5-4x1)+4(3x1-2x4)

= 9X9-6x11+4x(-5) = 81-66-20 = -5 Ans

Q4. Solve the Equation bycramersrule $9x_1 + x_2 = 13$ and $8x_1 + 2x_2 = 16$

Sol. $9x_1 + x_2 = 13$

$8x_1 + 2x_2 = 16$

$A = \begin{bmatrix} 9 & 1 \\ 8 & 2 \end{bmatrix}$ x= $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ and B= $\begin{bmatrix} 13 \\ 16 \end{bmatrix}$

$|A| = \begin{bmatrix} 9 & 1 \\ 8 & 2 \end{bmatrix}$ =9x2-8x1 = 18 – 8 = 10

$|A_1| = \begin{bmatrix} 13 & 1 \\ 16 & 2 \end{bmatrix}$ =13x2-16x1 = 26 – 16 = 10

$|A_2| = \begin{bmatrix} 9 & 13 \\ 8 & 16 \end{bmatrix}$ =9x16-8x13 = 144 –104 = 40

X_1 =$|A_1|/|A|$ = 10/10 =1 Ans

X_2 =$|A_2|/|A|$ = 40/10 =4 Ans

Q5: Solve the following linear equation by cramer's rule x + y + z =1, x + 2y + 3z =6 and x + 3y + 4z =6

Sol. x + y + z =1

x + 2y + 3z =6

x + 3y + 4z =6

A = $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ x= $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and B= $\begin{bmatrix} 1 \\ 6 \\ 6 \end{bmatrix}$

$|A|$ = $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 4 \end{bmatrix}$

Or $|A| = 1\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$ -1 $\begin{bmatrix} 1 & 3 \\ 1 & 4 \end{bmatrix}$ +1 $\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$

$|A|$= 9(8-9) -1(4-3) +1(3-2) = -1 -1 +1 = -1

$|A_1|$ = $\begin{bmatrix} 1 & 1 & 1 \\ 6 & 2 & 3 \\ 6 & 3 & 4 \end{bmatrix}$

Or $|A_1| = 1\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$ -1 $\begin{bmatrix} 6 & 3 \\ 6 & 4 \end{bmatrix}$ +1 $\begin{bmatrix} 6 & 2 \\ 6 & 3 \end{bmatrix}$ = -1(8-9)-1(24-18)+1(18-12) =-1 -6 +6 = -1

$|A_2|$ = $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 6 & 3 \\ 1 & 6 & 4 \end{bmatrix}$

Or $|A_2| = 1\begin{bmatrix} 6 & 3 \\ 6 & 4 \end{bmatrix} -1\begin{bmatrix} 1 & 3 \\ 1 & 4 \end{bmatrix} +1\begin{bmatrix} 1 & 6 \\ 1 & 6 \end{bmatrix}$ = 1(24-18) -1(4-3) +1(6-6) = 6 -1 +0 = 5

$|A_3| = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 6 \\ 1 & 3 & 6 \end{bmatrix}$

Or $|A_3| = 1\begin{bmatrix} 2 & 6 \\ 3 & 6 \end{bmatrix} -1\begin{bmatrix} 1 & 6 \\ 1 & 6 \end{bmatrix} +1\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$ = 1(12 -18) -1(6 -6) +1(3-2) = -6-0 +1 = -5

There fore X = $|A_1|/|A|$ = -1/-1 =1 Ans

Y = $|A_2|/|A|$ = 5/-1 = -5 Ans

z= $|A_3|/|A|$ = -5/-1 = 5 Ans.

END