Topic: - “INDEX NUMBER AND TIME SERIES ANALYSIS”

Prepared by

(Dr. Sabina Ahad)

Assistant Professor (Economics)
Government Degree College for women
Nawa Kadal, Srinagar.

Meaning of Index number:

An index number is a statistical device designed to measure any relative changes in the level of a phenomenon (variable or a group of variables) with respect to time, geographical location or other characteristics. The phenomenon or variables under consideration may be:

I. The price of any commodity like wheat, rice, milk etc or a group of commodities like consumer goods, producer goods, cosmetics etc.
II. The volume of industrial of agricultural production, exports, imports, trade etc.
III. The national income of the country, per capita income, foreign exchange reserves etc.

With the help of index numbers we can measure the average change in such phenomenon over a period of time or at different places. For example, suppose we want to study the general change in the price level of consumer goods during a period of time. These are not directly measurable as the price quotations of different commodities are expressed in different units e.g. Prices of wheat, rice, pulses etc. are quoted in Rs. per quintal, price of water is quoted in Rs. per gallon, prices of milk, petrol, kerosene etc are quoted in Rs. per litre, price of cloth in Rs. per meter and so on. Again, the prices of some commodities may be increasing while those of other commodities may be decreasing over the time period under consideration. Again, the rate of increase or decrease may be different for different commodities. Thus, it is not possible to directly measure the changes in prices and find the average rate of change. Index numbers enable us to arrive at a single representative figure which tells us about the average change in the price level of consumer goods. Thus, index numbers are called specialized types of average. They are called specialized type of rates, ratios, percentages which give the general level of magnitude of a group of distinct but related variables in two or more situation.

Definitions:

"An index number is a statically measure designed to show changes in variables of a group of related variables with respect to time and geographic location of other characteristics."

-- Spriegel

"Index numbers are devices for measuring differences in the magnitude of a group of relisted variables."

-- CroxtonCowdon

"Index numbers are used to measure changes in some qualities, which we cannot observe directly."

--Dr. A. L. Bowley

Characteristics of Index Number:

An index number has the following characteristics:

1. Expressed in percentage.
2. Comparative measurement.
3. Measure for changes.
4. Specialized type of averages.

1. **Expressed in percentage:** Relative changes are expressed in terms of percentage. We never use sign (%). It is assumed.
2. **Comparative measurement:** Index Numbers are always comparative. They compare changes taking place in the two variables. It is relative measure. If the index number of 2002 is 160 as compared to 100 in 1995, it shows that prices in 2002 have increased by 60% as compared to 1995.
3. **Measure for changes:** Index Number is capable of measuring changes, which cannot be directly measured. It measures changes in magnitude, which are complex, composite and are not capable of being measured directly.
4. **Specialized type of averages:** Index Numbers helps us in comparing changes in the series, which are in different units, whereas averages can be used to compare only those series which are expressed in the same units. This is why, index numbers are called specialized averages.

Uses of Index Numbers: Index numbers are today one of the most useful statistical devices. It is difficult to find any field of quantitative measurement where index numbers are not used. They are used in almost all sciences- Natural, social and physical. The main uses of index numbers are discussed below.

1. **Index numbers are economical barometers.** Like barometers which are used in physics and chemistry to measure atmospheric pressure, index numbers are rightly called as economic barometers which measure the pressure of economic and business behavior. In the words of Simpson and Kafka, "Index numbers are used to take the pulse of economy and they have come to be used as indicators of inflationary or deflationary tendencies." A careful study of index numbers of national income, prices, imports, export, consumption, production-Agricultural Industrial etc gives us a fairly good appraisal of the general trade, economic development and business activity of the country.
2. **Index numbers are helpful in framing suitable policies.** Many economic and business policies are based on the information provided by index number. For example, Dearness Allowance is fixed by considering the cost of living index number. Similarly, requirements for raw materials, labour, electricity etc are determined on the basis of indices of industrial and agricultural production. Import and export indices dictate various policies in the field of foreign trade. The excise duty on the production and sale of a commodity is determined according to the index numbers of the consumption of the commodity from time to time and so on.
3. **Index numbers measure the purchasing power of money.** There exists an inverse relation between the value of money and the price level. By using price index we can very easily calculate the power of money at any place and time as follows:

   Purchasing power of money = 1/Price index

   For example, if the consumer Price Index in any year is 150, then the purchasing power of one rupee will be 1/160 = 62.5 paisa.
4. **Index numbers are very useful in deflating.** We know that the Purchasing power of money goes on changing with the change in price level. Index numbers are used to adjust the

original data for price changes and thus convert nominal wages into real wages or nominal income into real income.

5. **Index numbers are helpful in studying trends and tendencies.** Since Index numbers study the relative changes in the level of any phenomenon over a period of time, they can be used to study the trend of the phenomenon in a time series data. For example, by examining the index number of the exports of India for last 10 to 12 years we can say about the trend of exports of India. Similarly, trends of national income. prices, consumption, balance of payments, terms of trade, wages, production etc can be formed by using index numbers.

Types of Index numbers:

Broadly speaking, there are three types of Index numbers:

1. Price Index Numbers.
2. Quantity Index Numbers.
3. Value Index Numbers.

1. **Price Index Numbers.** Price Index numbers show the changes in prices of their commodities produced or consumed in a given period with reference to some base period. Price indices can be of two types:
   I. Wholesale Price index numbers. They show the changes in the general price level of a country.
   II. Retail Price index numbers. They reflect the changes in the retails prices of commodities like wheat, rice, milk, cosmetics etc. The cost of living index is a special kind of retail price index.
2. **Quantity price Index Numbers.** These index numbers measure changes in the quantity of goods produced, consumed or distributed during the current period with reference to any base period.
3. **Value Index Numbers.** These Index numbers are prepared to compare changes in the value of any phenomenon, in the current period with reference to any base period. For example, changes in total revenue (Price x Quantity) can be studied by constructing a value index number.

Problems in the construction of Index numbers. While constructing index numbers, various problems are faced by the statisticians. These are also called the preliminaries to the construction of index numbers. These are given below:

1. **The Purpose of the index Number.** The first and foremost task before constructing any index number is to define in clear terms the objective or purpose of index numbers. There is no all-purpose index number. The knowledge about purpose of index number helps us to collect relevant data, select appropriate commodities, assign suitable weights and use proper techniques. Failure to decide clearly the purpose of the index would lead to confusion and wastage of time with no fruitful results e.g. Suppose we want to study changes in cost of living. The class of people for whom index is to constructed must be clearly defined so that we can start the task of preparing relevant index number accurately.

2. **Selection of Commodities.** After defining clearly the purpose of index, next problem is to make a selection of commodities which are to be included in the index. The selected commodities should be representative of the tastes, habits, customs etc of the people and they must be of standard quality, neither too large nor too small. Thus, in a consumer price index for people belonging to lower income groups, commodities like car, VCR, color TV should not be included. On the other hand, if the index number is for richer class of people, then index number will not be representative if we exclude above mentioned commodities.
3. **Obtaining Price Quotations.** After the commodities have been selected and their number decided, next problem is to obtain price quotations for these commodities. The data relating to prices of selected commodities may be collected from standard trade journals, reputed periodicals, news-papers and government publications. The collected data must be suitable to the purpose of the index. Also prices of commodities differ from place to place and shop to shop. After deciding the place and shop from where prices of commodities are to be taken, the next job is to appoint an authority who will supply the price quotations from time to time on regular basis. In order to check the accuracy of price quotations supplied by an agency the price quotations can be obtained from more than one agency.
4. **Selection of Base Period.** Whenever we construct an Index number, we construct it with reference to some base period. The index for base period is always taken as hundred. The following points need careful consideration regarding base period:
   I. The base period should be a normal one.
   II. The base period should not be too distant in the past
   III. Fixed base or chain base
5. **Selection of appropriate weights.** By weight we mean the relative importance given to different items. There are two types of Index numbers. I) Un-weighted index numbers. II) Weighted index numbers. In the first category equal importance is given to all the items while in the second category weights are assigned to different items depending upon their importance. While constructing weighted price index numbers, the quantity is takes as weight and while constructing weighted quantity index numbers, prices of different commodities are taken as weights.

   There are two methods of assigning weights:

   (i)Implicit weights and (ii) Explicit weights. Implicit weight is a method of giving varying emphasis to different items by the number of times a given item is included in the index whereas in the case of explicit weight, some outward evidence importance of various items in the index is given.

   Again, there can be either quantity weights or value weights. When aggregative method is employed for constructing index numbers, then quantity weights are used. On the other hand, when average of relatives' method is used for constructing index numbers, then value weights are used.
6. **Selection of Average.** Another problem which generally arises in the construction of index numbers is the choice of numbers. In the construction of index numbers, first price relatives are obtained. These price relatives have to be averaged to find one representative value of the index. For this purpose, arithmetic mean, median, mode, geometric mean, harmonic mean are

available. But median, mode and harmonic mean are never used. The choice lies between arithmetic mean and geometric mean.

Theoretically the geometric mean is considered to be the most appropriate average in the construction of index numbers because it gives equal weights to the equal ratios of change, more importance to small items than large items and satisfy the time reversal test and the factor reversal test.

7. **Selection of an Appropriate formula.** A large number of formulae have been developed by different statisticians for constructing Index numbers. Thus there is one more problem of selecting an appropriate formula. The choice of the formula depends on it) the purpose of index ii) Weighted or un weighted iii) Aggregative or average of relatives iv) Use of average and v) the availability of data.

Various Notations and Terminology. Before studying the methods of constructing index numbers, it is important to know the meaning of various notations and terms used in their construction.

Base year. It is the year with reference to which the comparisons are made it is denoted by suffix '0'.

Current Year. It is the year for which comparisons are made. It is denoted by suffix '1'

P_0: Price of the commodity in the base year.

P_1: Price of the commodity in the current year.

q_0: Quantity in the base year

q_1: Quantity in the current year.

P:Price relative expressing current year price as percentage of base year price.

$P=P_1/P_0 \text{ x } 100$.

P_{01}: Price index number for the current year with reference to the base year.

P_{10}: Price index number for the base year with reference to the base year.

Qo_1 : Quantity index number for the current year with reference to the base year.

$Q1_0$: Quantity index number for the base year with reference to the current year

$V0_1$: Value index number in the current year with reference to the base year.

Methods of constructing Index number: -

There are two methods of constructing Index number as mentioned below:

1. Simple aggregative.
2. Simple average of price relatives.

1. **Simple aggregative method.** It is the simplest of all the methods of constructing price index number. In order to calculate index number through this method, following are the important steps which are required:

P_{01} : Price index number of current year
P_1 : Current year's price
P_0 : Base years price
ΣP_0 : Total of base year's price of difficult commodities
ΣP_1 : Total of current year's prices for different commodities

$P_{01} = \Sigma P_1 / \Sigma P_0 \times 100$

This can be understood with the help of an example

Commodities	A	B	C	D	E	F	
Price in1985 P_0	50	40	80	110	40	70	$\Sigma P_0 = 390$
Price in 1995 P_1	70	60	90	120	60	70	$\Sigma P_1 = 470$

Applying the values in formula:

$P_{01} = P_1/P_0 \times 100$
$P_{01} = \Sigma P_1/ \Sigma P_0 \times 100 \quad = 470/390 \times 100 = 120.5$

The price index number for 1995 is 120.5, which shows that net increase in the prices of commodities in 1995 as compared to 1985 has been to the extent of 20.5%.

Limitations of the Simple Aggregative method:

I. **Weightage not given:-**Though the items have their own significance varying in degrees. Their relative importance is different but simple aggregative method does not accord any weightage to the items.
II. **Greater influence of items with largest unit price: -** All the items do not influence equally. Items having larger unit price influence: ΣP_1 in their favour, so the index number is also affected by them.

2. **Simple average of Price relative Method:**

While calculating index number according to this method we shall be adopting the following steps:

I. Price relatives of the current year will be calculated by applying the formula $P_1/P_0 \times 100$
II. Total of the price relatives shall be obtained
III. Aggregate of the price relatives will be divided by the number of commodities.
IV. Finally we shall apply the following formula.

$$P_{01} = \Sigma(P_1/P_0 \times 100) / N$$

Here P_{01} = Price Index number of the current year

$P_1/p_2 \times 100$ = Price relatives of the current year.

N = number of commodities.

Index number according to this method is the arithmetic mean or median or geometric mean of collected price relatives.

The below mentioned example can be used to explain this.

Commodities	A	B	C	D	E	F
Price in 1987(P_0)	50	40	80	110	100	60
Price in 1998(P_1)	70	60	90	120	170	60
Price relatives P_1 / P_0 x 100	70/50x100 =140	60/40x100 =150	90/80x100 =112.5	120/110x100 = 109	170/100x100 = 170	60/60x100 =100

$P_{01} = \Sigma (P_1 /P_0 \text{ x } 100) / N$

Here P_{01} = Price index number of the current year.

$\Sigma\ P_1/P_0 \text{ x } 100 = 781.5$ (Aggregates of prime relatives)

N= 6 (Number of commodities)

$P_{01} = \Sigma (P_1 /P_0 \text{ x } 100) / N = 781.5 / 6 = 130.25$

It shows that the index number for the current year 1998 is 130.25. It means that the price for 1998 has increased by 30.25% as compared to the price for 1987.

Merits of Simple Average of Price Relative Method.

1. Not influenced by extreme items**:** This method of calculating index number is not influenced by extreme items. It gives equal importance to all the items.
2. It is not influenced by absolute level of individual prices: Index number calculated according to this method is not influenced by units in which price is quoted or absolute level of individual prices.

Limitations of Simple Average of Price Relative Method:

1. Assumption of equal relatives: It is assumed that relatives have equal importance, it may not be always true.
2. Problem of selecting proper average: As index number according tothis method is the division of total of price relatives by mean or median or geometric mean. It is difficult to make a choice between them as the most suitable average.

Test of consistency:

There is no perfect formula for measuring changes over time and no index number can adequately represent all the changes taking place from time to time. It is difficult to lay down any definite rule to be followed in the construction of a formula for an ideal index number.

Fisher, however, has suggested two tests, that should be met by a good index number formula. There is no mathematical basis of these tests of consistency and any conclusions drawn with their help about any formula need not to be considered as adequate or final.

1. Time reversal test:

The formula for the index number should be such that the product of the index number by another index number based on the same data with time interchanged should be equal to one.

If P_{01} is the price index number of the year one with base year 0 and P_{10} is the price index number of the year 0 with the base year 1 then the test requires that $P_{01} \times P_{10} = 1$ (ignoring multiplication by 100).

This test is not satisfied by L and P. Thus writing P_{01} for L in Laspeyre's formula, we have $P_{01} = \Sigma Pi\, q_0 / \Sigma P_0\, q_0$, $P_{10} = \Sigma P_0\, q_1 / \Sigma P_1\, q_1$

$$P_{01} \times P_{10} = \Sigma P_1 q_0 / \Sigma P_0 q_0 \times \Sigma P_0 q_1 / \Sigma P_1 q_1 \neq 1$$

Laspeyre's formula does not satisfy the time reversal test. Paesche's formula also does not satisfy this test. Fisher's index number satisfies time reversal test. Writing now P_{01} for F,

$$P_{01} = \sqrt{\Sigma P_1 q_0 / \Sigma P_0 q_0 \times \Sigma P_1 q_1 / \Sigma P_0 q_1}\ ,\ \sqrt{P_{10} \Sigma P_0 q_1 / \Sigma P_1 q_1 \times \Sigma P_1 q_1 / \Sigma P_1 q_0}$$

$$P_{01} \times P_{10} = \sqrt{\Sigma P_1 q_0 / \Sigma P_0 q_0 \times \Sigma P_1 q_0 / \Sigma P_0 q_1 \times \Sigma P_0 q_0 / \Sigma P_1 q_0} \quad = \quad 1$$

Note: If for any index number $P_{01}P_{10} \neq 1$, it may be interpreted to mean that there is a time bias or error in index formula given by $B_T = P_{01}P_{10} - 1$

2. Factor Reversal Test:

The formula for the index number should be such that the product of the index number by another index number based on the same data with price and quantity interchanged should be equal to the ratio is aggregate value in the current to the aggregate value in the base year.

If P_{01} is the index number for year 1 with year 0 as base and Q_{01} is the index number for year 1 with year 0 as base (obtained by interchanging p's and q's) then the test requires that

$P_{01} \times Q_{01} = \Sigma p_1 q_1 / \Sigma p_0 q_0$ = value in current year / value in base year = V_{01}

This biasis given by $B_F = P_{01}Q_{01} / V_{01} = 1$

L and P fail in this test. Thus, if we write P_{01} for L

$$P_{01} = \Sigma p_1 q_0 / \Sigma p_0 q_0,\ Q_{01} = \Sigma p_1 q_0 / \Sigma q_0 p_0$$

$$P_{01} \times Q_{01} = \Sigma p_1 q_0 / \Sigma p_0 q_0 \times \Sigma p_1 q_0 / \Sigma q_0 p_0 \neq \Sigma p_1 q_0 / \Sigma p_0 q_0$$

The factor reversal test is satisfied by Fisher's formula:

$$P_{01} = \sqrt{\Sigma P_1 q_0 / \Sigma P_0 q_0 \times \Sigma P_1 q_1 / \Sigma P_0 q_1}\ ,\ Q_{01} = \sqrt{\Sigma Pi\, q_0 / \Sigma P_0 q_0 \times \Sigma q_1 P_1 / \Sigma q_0 P_1}$$

$$P_{01}Q_{01} = \sqrt{\Sigma P_1 q_0 / \Sigma P_0 q_0 \times \Sigma P_1 q_1 / \Sigma P_0 q_1 \times \Sigma P_1 q_0 / \Sigma P_0 q_0 \times \Sigma q_1 P_1 / \Sigma q_0 P_1} =$$

$$\sqrt{\Sigma P_1 q_1 / \Sigma P_0 q_0 \times \Sigma P_1 q_1 / \Sigma P_0 q_0} = \Sigma P_1 q_1 / \Sigma P_0 q_0$$

Fisher's Index number is called an ideal index number because it satisfies both the time reversal and factor reversal tests.

3. A third test known as circular test is an extension of time reversal test. If
   P_{01} is the price index of year 1 with base year 0,
   P_{12} is the price index of year 2 with base year 1,
   P_{20} is the price index of year 0 with base year 2,
   Then $P_{01} \times P_{12} \times P_{20} = 1$ (with three periods)
   In general if there are n periods
   $P_{01} \times P_{12} \times P_{23} \times$ ----- $Pn_0 = 1$
   This test is not satisfied by most index numbers.

Other tests of Index number Formulae:

Index numbers may be regarded as indicators of a change in a group of statistical series allowing us to split the total change into its components called sub – Indices I_1, I_2 ----, I_n. The total index I is the product of the sub- indices:

$I = I_1 \times I_2 \text{---} \times I_n$.

4. **Test of Proportionality:** If all sub-indices are equal to A the total index equals A.
5. **Test of Definiteness:** If one of the Sub-indices is zero or infinity, the total index must not become zero or infinity or indeterminate.
6. **Test of commensurability:** If the unit of measurement is changed then the value of the index must not change.

Chain base Index Numbers:

This technique provides the comparison of any given year's price with the previous year's price.

Each year's price is taken as the base in the next year. Thus 1941 is the base for 1942 (here the 1st year 1941 = 100), then 1942 is the base for 1943 etc.

a) The Chain Base or Link index number (CBIN) gives a relation with the immediate past rather than a distant past.
b) This avoids seasonal variation since the time gap is narrowed down.
c) Outdated or unwanted items can be dropped and new items added if required.

Thus, the base year is not fixed but changes from year to year. For example, for 1980, 1979 will be the base; for 1981, 1980 will be the base and so on. The relatives obtained are known as Link Relatives or Link Index Numbers. These link relatives are chained together by successive multiplication to get a chain index.

Steps for constructing Chain Index:

I. First we find link relatives by expressing price for each year as percentage of proceeding year.
   Link Relative = Current year price / Previous year price x 100

II. These Link Relatives are chained together by successive multiplication to form a chain index. The following formula is used

$$\text{Chain Index for current year} = \frac{\text{Current year Link Relative x Chain Index of proceeding year}}{100}$$

III. The procedure for conversion from Fixed base (FB) to Chain base (CB) can be seen below:

Year -	1964	1965	1966	1967	1968
Price	75	50	65	60	72

Solution:

calculations for Chain Index

Year	**Price**	**Link Relatives**	**Chain Base Index Numbers**	**Fixed Base Index Numbers (1964 = 100)**
1964	75	100	100	100
1965	50	50/75x100 =66.67	66.67x100/100 =66.67	50/75x100 =66.67
1966	65	65/50x100 =130	130x66.67/100 =86.67	65/75x100 =86.67
1967	60	60/65x100 =92.31	92.31x86.67/100 =80	60/75x100 =80
1968	72	72/60x100 =120	120x80/100 =96	72/75x100 =96

Shifting and splicing of Index numbers:

As the name itself suggests the concept is giving explanation about shifting the base, so far we have studied the construction of index numbers by using a fixed base, i.e. keeping the base period fixed throughout the computation of index number. Contrasted with it is the shifting base method in which relatives for each year are calculated upon the prices of the preceding year. Thus, the base year is not fixed but changed from year to year.

This can be achieved by using the following relation as:

$$\textbf{Shifting Price Index} = \frac{\text{Original Price index}}{\text{Price index for new base year}} \times 100$$

Explanation:

e.g. for the years 1981,1982, 1983, 1984

Old Index Number (base year 1981) 120, 150, 180, 225.
Here we have to find out new index number (base year 1984). This can be done:

New Index number = Original Price Index / Price Index for 1984 x 100

= 120 / 225 x 100 = 53.33, 150/225 x 100 = 66.66 ,
180/ 225x100 = 80, 225/ 225x100 = 100

Splicing: Two series of Index numbers

An old index number series may be discontinued because of obsolete items included in it or other reasons. If a new series is constructed with a discontinuation year of the first as base, thus two series so obtained, having different bases are not comparable. But the two series can be spliced together into one continuous series by multiplying each Index number of the new series by the Index number of the discontinuation year of the old series. In other words, the process combining two or more index numbers covering different bases into a single series is known as splicing. For instance, we have two series i.e. X series and Y series, and if old series is spliced with new then backward splicing = Index X of current year / Index X of common year x 100.

If series Y spliced with X then Forward splicing = Index Y of current year x Index Y of common year / 100

For Instance

Year	**X Series Col (1)**	**Y Series Col (2)**	**Spliced Series x Base Col(3)**	**Spliced Series y Base Col(3)**
1951	100	-	100	50
1952	160	-	160	80
1953	200	100	200	100
1954	-	90	180	90
1955	-	105	210	105
1956	-	165	210	105

Explanation:

Year	Col.3 X Series	Col.4 Y Series
1951	100	100/200x100=50
1952	160	100/200x160=80

1953	200	100/200x220=100
1954	200/100x90= 180	90
1955	200/100x110=220	110
1956	200/100x105=210	105

Deflating:

Deflating is a technique which can be used to make allowances for the effect of changing price value. To deflate a value, divide it by the Index number and multiply by 100. The deflated values represent the purchasing power. It is used to measure the purchasing power of money. It may be noted that changes in real P.C.I are small as compared with those in money incomes showing that although money incomes seem to have increased, purchasing power has only marginally increased.

$$\text{Deflated Value} = \frac{\text{Current value}}{\text{Price index of the current year}} \times 100$$

Explanation:

Year	**Income (In crores)**	**Price Index**	**Deflated (real) P.C.I**	**Explanation**
1951	120	100	120	120/100x100=120
1952	140	110	127	140/110x100=127
1953	150	120	125	150/120x100=125
1954	180	140	129	180/140x100=129
1955	200	150	133	200/150x100=133

Time Series- **(Section-II)**

Meaning:-

A time series is a sequence of observation on a variable mode at regular points or intervals of time and arranged in chronological order. The importance of regularity and order in which the observations are made is explicitly recognized.

Economic and business activities provide us with various examples of time series i.e. prices, production, employment and National Income data, noted at regular intervals of time, are families cases of Time Series.

It is convenient to represent a time series graphically either by joining the points by straight lines or by a smooth curve in either case the major characteristics of the time series becomes evident.

Definition:-

Lincoin L, Choo… "A time series is set of quantitative readings of some variable or composite of variables arranged in chronological order of their occurrences."

W.J. Reichman….. "A time series is simply a set of recordings of value of variable quantities measured at intervals of time. The base quantity of the data from which such series are divided is one of historical fact."

Utility of Time Series:

1. It helps in Planning Future Operations:- The analysis can be used to forecast the probable economic events in the near future based on past knowledge. Budgeting and Planning of economic activities depend considerably on accurate economic forecasts.
2. It helps in Understanding Past Behavior:- The analysis can be used to control the process producing the services. For instance, it may suggest method to check inflationary forces before they raise their heads.
3. It helps in everlasting current accomplishments:- Time series analysis can help in understanding the mechanism generating the series. It can be useful in the description of the features of an economy.
4. It facilitates Comparison: Time series analysis in economics could give an explanation of the forces of inflation and depression.

Components of Time Series Analysis:

A time series is the result of simultaneous interaction of various forces, some recurring and non-recurring others non-recurring at intervals. Many theories have been put forth to explain various types of fluctuations, particularly trade cycles, arising in economics and other sciences. An analysis of time series can test the validity of these theories.

There are four major types of variations or components of a time series:

I. Secular Trend T

II. Cyclical variations C

III. Other periodic changes, particularly Seasonal changes S

IV. Random or Irregular movements R.

All these four components, jointly are severely, are found in a time series.

T: Secular Trend:

The trend is the general, smooth , long term movement of the series. It is a gradual movement over the whole period showing a persistent growth, decline, or any other long run tendency**.**

The trend ignores sudden or short term fluctuations. It moves steadily in a certain direction for a fairly long period of time. The term long period is relative. One year or even a month may be a long period in some cases. In other cases, as in agriculture, even fifty years may be inadequate some times. The movement may be slow or fast.

Populations trends often show an upward movement. Death rates may show a downward trend.

It is possible that upward and downward movement as well as stability may be noticed in different part of a trend.

Time trend relative to the period of reference. Changes in population, technology capital formation and other factors give a shape to the trend. Social, economic and political institutions are the basic elements underlying the trend.

C: Cyclical variations:

These are long term movements which represent constantly recurring prices and falls in the variable under consideration. Fluctuations over a period of one or more years nearly regular intervals in economic activity give rise to business cycle which show alternating periods of depression and prosperity in the economy. Cycles of economic variables like wages, employment, prices, production etc are the result of interaction between and the cumulative act of political, economic, social and other forces.

The amplitude and the period of the oscillating cycle may not be very regular but a perspective view of the graph of time series in general shows a more or less regular pattern of cycles recurring roughly every three, four are more years and sometimes even eighteen years.

An observation y_1 of as time series is a peak ifits value is greater than the immediately preceding (y_1 – 1) and immediately showing (y_t+1) observation.

For a peak $y_t – 1 < y_t > y_t +1$

For a trough $y_t -1 > y_t < y_t +1$

Phase: The interval between two successive turning points is a phase of the time series.

Period: The interval between two successive peaks (or troughs) is the period of oscillation. One complete period gives rise to a cycle which is a four phase cycle made of, for instance, a boom, a recession, a depression and recovery in a business cycle.

S: Seasonal Changes:

These are variations that occur regularly at a specific intervals of time. Seasonal movements are cycles of relatively short duration, mostly one year and related to seasonal factors and customs.

Every year seasons follow practically the same pattern and seasonal variations follow the seasons, so do the customs and other rhythmic forces.

Rainfall, temperatures, eggs laid by hens, harvest, certain types of investments etc are examples showing seasonal variations.

Seasonal variations occur due to natural forces. They may also occur as a result of manmade customs, rites, festivals and conventions.

Ina time series, seasonal variations may sometimes be misinterpreted to mean other types of variations. For a better understanding of the components of a time series, the data should be corrected for seasonal variation i.e. the series should be de-seasonalized writing y_t / S_t or y_t – S_t (depending on the assumption of multiplication or additivity of components). Here S_t stands for seasonal variations. Seasonal variations are considered either for their own sake or for being removed, thus allowing us to concentrate on the remaining variation. Production or purchase of certain commodities take place according to the nature of the season as in the woollens in winter or umbrellas in the rainy season.

R: Random Movements:

These occur in a completely unpredictable manner. They do not follow any definite pattern like the trend, cyclical or seasonal movements. Irregular variations are strictly random movements which keep on operating all the time. Episodic or catastrophic variations occur from time to time in the form of strikes, wars, floods, political events etc. The minute to minute changes in the prices of stocks give us good example of random movements.

Analysis of Time series:

The Objective of time series analysis is to discover the magnitude and direction of I) the trend that may exist in the series II) the nature and amplitude of the cycles III) the effect of seasonal changes and IV) the size of random movements.

Before proceeding with an analysis, often, corrections in the time series for changes in various factors like population, prices etc are made.

The four components of a time series are not necessarily mutually independent. Any one of them may be under the influence of one or more of the remaining forces thus complicating the work of analysis.

Sometimes one or more of these components may not be recognized owing to a mix up lack of regularity in the components may be highly confusing.

The Trend:

A study of the trend may useful in itself and also in the operation of deviations of periodic movements. It may be useful for comparison with the past and present trends and particularly helpful in having a peep into the probable future i.e. forecasting.

The trend may be determined by the following methods:

(i) Free hand drawing
(ii) The method of moving averages.
(iii) High and low midpoint method
(iv) The method of semi-averages
(v) The method of mathematical functions

(i) Free Hand writing: To get the general nature of the trend in a time series, a smooth curve may be drawn through the graph of the series.

The method is simple, quick and useful in determining any kind of trend, linear or otherwise but even an expert with a sound judgment many not be able to give an exact graphical trend for any data. The result is rough, approximate and subjective.

(ii) The method of moving averages: Consists in finding the average of a certain number of terms of the time series and taking this average as the trend value for the middle of the period covered in the calculation of the average which is the period of the moving average. Successive moving averages are calculated for the data by omitting every time the beginning observation and adding the one immediately after the last.

A moving average is useful in smoothing out periodic fluctuations of the cyclical type, particularly when the period of the moving average is carefully selected.

The longer the period of the moving average the smoother is the trend likely to be provided the period coincides with the period of the cycle.

The method of the moving averages can also be used in the context of seasonal variations where the period if 12months or 4 seasons or some other division of the year. It may be noted that the method of the moving averages has a descriptive rather than a analytic value

Period of moving average:

If the period of the moving average is equal to the period of the cycle or is the multiple of it, the smoothing is perfect and straight and if the period of the cycle is not fixed because of fluctuations, the trend will be a curve.

The moving may remove the curvature if any, in the original series. The moving average cannot remove such fluctuations in a series as are irregular throughout. A moving average with a longer period can remove these fluctuation to a greater extent.

The method of moving averages cannot be applied with success to every series but when it is successfully applied the trend may be stretch free hand on either side for prediction purposes.

(iii) The High Low Midpoints Method:

In this method we connect by straight lines the high points of each cycle and determined by interpolation the value on these lines for each year of the times series. The same is done with the low points. The average of each pair of high and the corresponding low points is obtained by taking the midpoints.

(iv) The Method of semi-averages:

The time series is divided into two or more confident, preferably equal parts for each of which an average is found. The lines joining the points corresponding to these averages gives the trend. The method should be applied only when the trend is linear or approximate linear.

If there is an even number of years say 10 years from 1961 to 1970, the two parts will be made of 1961 – 65 and 1966 – 70.

If there is a odd number of pairs as in 1961-71 omit the middle year 1966. The two parts are 1961 – 65, 1967 – 71.

Compute the arithmetic mean of each point and plot against the central year of each part. The line joining the two points is the required trend. The linearity of trend is an assumption which may not necessarily hold in practice.

(v) The Method of Mathematical Functions:

Describes a tend as well as shows the law of development of a time series. A mathematical function is very useful for prediction purposes though the selection of a proper function can be different.

Several simple functions are available, one of the simplest is a straight line function: $y = a+bx$ where time is represented by x which is the independent variable and y is the dependent variable. The values of parameters a, b are determined by the principle of least squares as explained on regression.

The trend is given by the regression of y on x, estimated by the principle of least squares: $y = 8+1.3x$

To compute the trend values substitute the value of x corresponding to years.

1961 – $x = -2$	$y=8+1.3(-2) = 5.4$
1962 – $x= -1$	$y=8+1.3(-1) = 6.7$
1963 – $x=0$	$y=8+1.3(0) = 8.0$
1964 – $x=1$	$y=8+1.3(1) = 9.3$
1965 – $x=2$	$y=8+1.3(2) = 10.6$

The computed trend values and short term fluctuations obtained by subtracting the trend values from the observed values are shown in separate columns.

Can we forecast the future trend?
We can forecast the trend though fo a not very remote period.

Thus for 1966, x=3 and y=8 +1.3 (3) = 11.9
for 1970, x=7 and y=8 +1.3 (7) = 17.1 etc.

In fitting a straight line trend to time series data in the above problem we have taken time deviations x=t – 1963. This is done for convenience in calculations.

The rule that should be followed is that when the number of years is odd, take middle year as the origin, like 1963 here so that the x – values are -2, -2, 0, 1, 2. This results in $\Sigma x=0$.

If the number of years is even, say 6, the origin should be taken at the middle of the two years i.e. the values are -5, -3, -1, 1, 3, 5. This also results in $\Sigma x=0$.

Non linear trends: A linear trend may not always be good fit to the data. Often the rise and fall in the data may be better expressed by a parabolic trend of the type:

$$Y = a+bx+cx^2$$

which is a second degree curve. We can find the values of a,b, c by solving the normal equations obtained by the principle of least squares.

$$\Sigma y = na+\Sigma x + c\Sigma x^2$$
$$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3$$
$$\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^2 + c\Sigma x^4$$

Usually a straight line trend fitted to the data. If after plotting the data we notice the presence of a second degree, log-linear or any other trend we can still use the method of least squares which (a) is free from subjective bias (b) provides annual rate of growth in a linear trend and (c) can help predict the values of the variable for any period in future or over the time series.

Note: This method ignores the presence of cyclical seasonal and random fluctuations in the data and is based on long term variations in the trend. It is sensitive to any change in the data.

If we have taken the time deviations according to the rule given above i.e. if x = t – (middle of all years) then
$\Sigma x = 0$, $\Sigma x^3 = 0$.
These normal equations take the simple form.

$$\Sigma y = na+c\Sigma x^2$$
$$\Sigma xy = b\Sigma x^2 \text{ or } b = \Sigma xy/ \Sigma x^2$$
$$\Sigma x^2 y = a\Sigma x^2 + c\Sigma x^4$$

The first and third equations give the values of a, c.

Many other types of trends can be fitted to time series data depending on the pattern of their movement. Non linear trends can usually be reduced to linear forms by suitable transformations for convenience in calculations.

Additive and multiplicative models:

Since we have already discussed the components of index numbers, we may assume an additive or multiplicative relationship among these components, which are

I. Secular Trend T

II. Cyclical variations C

III. Other periodic changes, particularly Seasonal changes S

IV. Random or Irregular movements R.

In the first case, the observed value (Y_t) at any time is assumed to be the resultant sum of the impact of four forces:

$Y_t = T + C + S + R$

In the second case, the resultant product of the play of all the four forces:

$Y_t = T \times C \times S \times R$

The multiplicative hypothesis amounts to a long additive relation. Since,

$Y_t = T \times C \times S \times R$

$\rightarrow$ Ln Yt = lnT+lnC+lnS+lnR

Where ln represents log.