**CONTROL CHART FOR ATTRIBUTES:**

Many quality characteristics can not be conveniently represented numerically. In such cases, we usually classify each item inspected as either conforming to the specifications on that quality characteristic or non-conforming to those specifications. The terminology defective or non-defective is often used to identify these two classifications of product quality characteristic is taken as an attribute and control charts employed there are called control charts for attributes. Moreover, often it is desirable to classify a product as either defective or non-defective on the basis of comparison with a standard. This classification is usually done to achieve economy and simplicity in the inspection operation. For example, the diameter of a ball bearing may be checked by determining whether it will pass through a gauge consisting of circular holes cut in a template. This kind of measurement would be much simpler than directly measuring the diameter with a device such as a micrometer. Control charts for attributes are used in these situations. Attribute control charts often require a considerably larger sample size than do their variable measurements counterparts.

**P-CHART OR CONTROL CHART FOR FRACTION DEFECTIVES:**

P-Chart, also known as the fraction or percent defective chart, is commonly used in dealing with attribute data to monitor the quality of a manufacturing process. In many situations, it is not necessary to find the variation pertaining to certain measurable characteristics. But the units are checked to see whether they possess certain attributes or not. For instance, whether the ball-bearings produced by a factory are perfectly circular, whether the iron shots are perfectly spherical. If not, they are defective. If the unit is defective, it can not be used and if non-defective, then it is acceptable. Moreover, if a lot contains a proportion of defective more than a certain percentage, it is not to be marketed. In case, more defective items come in the market for sale, the reputation of the company will go down and the product may lose the market. Hence, the process is tested to see whether it is under control with regard to fraction defectives.

Suppose P is the population fraction defective, in a process during a fixed i.e, in a lot of items produced during that fixed period, the judgment about the process is based on the fraction defective, p, in a sample of size n. Let the sample contains d defectives. Hence $p = d/n$ also,

To test whether the process is under control or not, we construct a p-chart. For this, samples of items are taken at regular intervals or from sub-groups and inspected. The proportion of each defectives in each sample is calculated. In this chart, p-values are taken on the Y-axis and the sample numbers on the X-axis. Since, an unit can either be defective or non-defective, the variable d is dichotomous and will follow binomial distribution. i.e d~ B( n, p)

Hence E(d)= nP and V(d)=nPQ, therefore, E(p)=E(d/n)=P or E(d)=nP and S.D(p)= S.D (d/n)= √P.Q/n where Q=1-P.

Therefore control limits for p-chart when standard are given become as

UCL=P+3 √PQ/n, CL= P and LCL= P-3√PQ/n

When the process fraction defective P is not known, then its estimate is obtained form thr data. The usual procedure is to take or select k samples each of size n from a lot. Let $d_i$ (i=1,2….k) be the defectives in k samples, then

$\hat{P} =$ Total no. of defectives in k samples/Total no. of units in k samples

= $d_1 + d_{2+ \ldots\ldots\ldots +} d_k/n+n+\ldots\ldots.+n = np_1+np_2+\ldots\ldots.np_k/nk=\bar{p}$

The statistic $\bar{p}$ estimates the unknown process fraction defective P.

Therefore, control limits for p-chart when P is not known are:

UCL= $\bar{p} + 3\sqrt{\bar{p}(1-\bar{p})/n}$ , CL= $\bar{p}$ and LCL= = $\bar{p} - 3\sqrt{\bar{p}(1-\bar{p})/n}$

Sample p-values are plotted on the graph against sample numbers. If any point lies outside the control limits, it is concluded that the process is out of control, otherwise not. The sample points which lie outside the upper control limits are called high spots and indicate the deterioration in the production process. Such a situation should immediately be reported to the production in-charge. Again, the points which fall below the lower control limits are called as low spots and indicate an improvement in the production process or give greater assurance of the good quality of the product. But, it should be confirmed that there is no slackness on the part of the checker.

P-CHART FOR VARIABLE SAMPLE SIZE:

The some applications of the control chart for fraction non-conforming the sample is a 100% inspection of process output over some period of time. Since different number of units could be produced in each period, the control chart would then have a variable sample size. There are several approaches to construct and operate a control chart with variable sample size.

## I. P-CHART WITH VARIABLE CONTROL LIMITS:

The most simple approach is to determine control limits for each individual sample that are based on the specific sample size. In this case an estimate of P is given by:

$\hat{P} =$ np$_1$+np$_2$+…….n p$_k$/n$_1$+n$_{2+}$…………n$_k$ = $\frac{\sum n_i\ p_i}{\sum n_i}$

Therefore, control limits are:

UCL= $\bar{p} +\ 3\sqrt{\bar{p}(1-\bar{p})/n_i}$ , CL= $\bar{p}$ and LCL= = $\bar{p} -\ 3\sqrt{\bar{p}(1-\bar{p})/n_i}$, where $n_i$ is the ith sample size.

### II. P- CHART WITH CONSTANT CONTROL LIMITS:

This approach is to base the control chart on average sample size resulting in approximate set control limits, thus assumes that the future sample size will not differ greatly from the previous sample. Thus if $n_1, n_2 \ldots\ldots n_k$ are k- samples, then their average sample size $\bar{n}=\frac{1}{k}\sum n_i$ is used for all samples. Therefore, approximate control limits are :

UCL= $\bar{p} +\ 3\sqrt{\bar{p}(1-\bar{p})/\bar{n}}$ , CL= $\bar{p}$ and LCL= = $\bar{p} -\ 3\sqrt{\bar{p}(1-\bar{p})/\bar{n}}$,

### III. STANDARDIZED P-CHART:

Another solution to the problem is to standardize the control chart, such type of control chart has central line at zero and UCL & LCL at +3 and – 3 respectively. Here, we compute value of standard variate for each sample corresponding to p- values and is given by:

P= P- E(P)/S.E(P) ~ N(0,1)

$$= \frac{P-\bar{p}}{\sqrt{\frac{\bar{p}\ (1-\bar{p}}{n}}} \sim N(0,1)$$

## CONTROL CHART FOR NUMBER OF DEFECTIVES (np-CHART):

The basic idea of np-chart is to see number of defective items does not exceed the permissible limits. If we take a random sample of size n from the lot randomly and let d denote the no. of defectives in the sample, then

P= no. of defective in the unit/ Total no. of units in the sample= d/n

Therefore, d= np and the occurrence of defective item follow binomial distribution, thus we can write, E(d)= nP and V(d)= nPQ, which gives the control limits as:

UCL= nP + 3√nP(1-P), CL= nP and LCL= nP -3√nP(1-P)

When P is unknown, then the unbiased estimate of P is obtained from k samples each of size n as $d_i$, i=1,2….k denotes the no. of defectives in the ith sample, then

$P_i = d_i/n$, i=1,2…k

$$\bar{P} = \frac{\sum d_i}{\sum n} = \frac{d_1 + d_2 + \cdots d_k}{n + n + \cdots nk} = \frac{np_1 + np_2 + \cdots np_k}{nk} = \frac{\sum p_i}{k}$$

Therefore, control limits for np-chart become:

UCL= $n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})}$, CL= $n\bar{p}$ and LCL= $n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}$

For the construction of np- Chart, d values for the samples are plotted against the sample numbers on the graph paper. If the plotted points lie within the control band, the process is considered to be under control and vice-versa.

**CONTROL CHART FOR NUMBER OF DEFECTS(C-CHART):**

There are many situations in industry where after classifying an item or product as a defective one, it is further examined for the number of defects contained in it e.g, a defective photography film may further be examined for the number of surface defects on it. Thus, every defective unit contains one or more of the defects. In fact, a defective item is that which does not conform to one or more of the specifications where as a defect is an instance of the articles lack of conformity to specifications. The number of defects is denoted by C. In a manufacturing process there are numerous opportunities for defects to occur but the actual occurrence of defects is quite casual. So the probability (p) for a defect to occur in any one spot of the product is negligible as compared with n which is the area opportunity here. So all this and otherwise also the experience with variation in the number of defects per unit indicates that the distribution of the variable follows the form of Poisson distribution.

If C is the average no. of defects per unit and our assumption is that it follows Poisson distn., then

$$P(X = c) = \frac{e^{-c}\, c^x}{x!}$$

E(X)= V(X)= C

Therefore, the control limits for C- Chart are given as:

UCL= C + 3 √C, CL= C and LCL= C - 3 √C

If C is not known, then its estimate can be obtained from K- samples as

$$\bar{C} = \frac{1}{k}\sum c_i$$

Therefore, the control limits for C-Chart are given as:

UCL= $\bar{C} + 3\sqrt{\bar{C}}$ , CL= $\bar{C}$ and LCL= $\bar{C} - 3\sqrt{\bar{C}}$

For the construction of C- Chart, c-values for the samples are plotted against the sample numbers on the graph paper. If the plotted points lie within the control band, the process is considered to be under control and vice-versa. C-chart can be used when the number of samples id from 20 to 25 and the value of c is round 5.

**APPLICATIONS OF C-CHART:**

I. It is used when a count of defects per article is considered and they are to be eliminated following 100% inspection.

II. It can be used to periodic samples where it is not necessary to resume all the defects but in order to get a good quality their number is kept low.

III. It can also be used in non-manufacturing process like:

a) In accident statistics, with c as the no. of accidents at any place at a given time.

b) In studying the effect of an epidemic disease, where c may be the no. of deaths per day caused by the disease in a particular region. In fact, it exhibits all the applications of the Poisson distribution.

Ex: To study the process of room readiness, sub-groups of 200 rooms were selected daily for a 4-week period. For each room in the sample, it was determined whether the room contained any non-conformances in terms of the availability of amenities and the working order of all appliances upon check-in. Data were collected on the non-conformances in 200 rooms. The following data list the number and proportion of non-conforming rooms for each day in the 4-week period. Draw the appropriate chart.

| Day | Rooms Studied | Rooms Not Ready | p |
|---|---|---|---|
| 1 | 200 | 16 | 0.080 |
| 2 | 200 | 7 | 0.035 |
| 3 | 200 | 21 | 0.105 |
| 4 | 200 | 17 | 0.085 |
| 5 | 200 | 25 | 0.125 |
| 6 | 200 | 19 | 0.095 |
| 7 | 200 | 16 | 0.080 |
| 8 | 200 | 15 | 0.075 |
| 9 | 200 | 11 | 0.055 |
| 10 | 200 | 12 | 0.060 |
| 11 | 200 | 22 | 0.110 |
| 12 | 200 | 20 | 0.100 |
| 13 | 200 | 17 | 0.085 |
| 14 | 200 | 26 | 0.130 |
| 15 | 200 | 18 | 0.090 |
| 16 | 200 | 13 | 0.065 |
| 17 | 200 | 15 | 0.075 |
| 18 | 200 | 10 | 0.050 |

| | | | |
|---|---|---|---|
| 19 | 200 | 14 | 0.070 |
| 20 | 200 | 25 | 0.125 |
| 21 | 200 | 19 | 0.095 |
| 22 | 200 | 12 | 0.060 |
| 23 | 200 | 6 | 0.030 |
| 24 | 200 | 12 | 0.060 |
| 25 | 200 | 18 | 0.090 |
| 26 | 200 | 15 | 0.075 |
| 27 | 200 | 20 | 0.100 |
| 28 | 200 | 22 | 0.110 |

Sol:

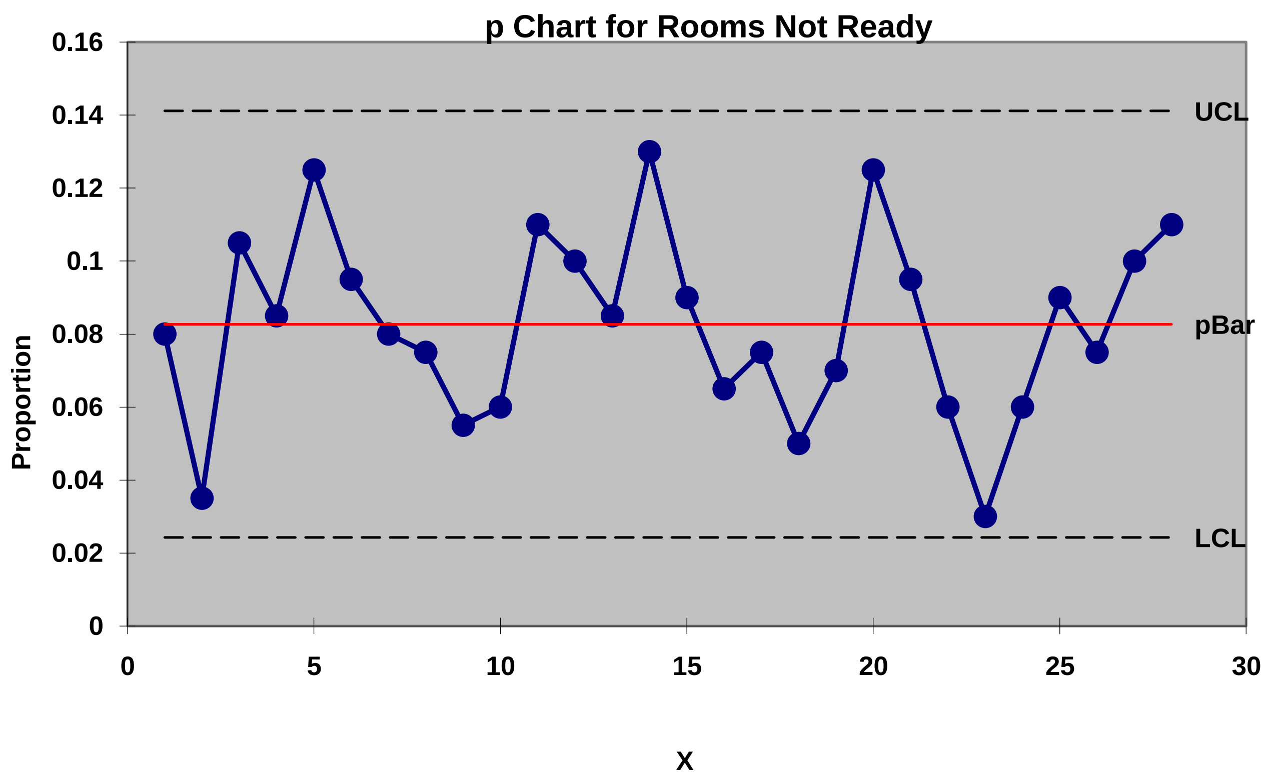
**p Chart for Rooms Not Ready**

| Intermediate Calculations | |
|---|---|
| Sum of Subgroup Sizes | 5600 |
| Number of Subgroups Taken | 28 |
| Average Sample/Subgroup Size | 200 |
| Average Proportion of Nonconforming Items | 0.08267857 |
| Three Standard Deviations | 0.05842026 |

| **p Chart Control Limits** | |
|---|---|
| **Lower Control Limit** | **0.02425831** |
| **Center** | **0.08267857** |
| **Upper Control Limit** | **0.14109883** |

| **Day** | **Studied** | **Rooms Not Ready** | **p** | **LCL** | **Center** | **UCL** |
|---|---|---|---|---|---|---|
| 1 | 200 | 16 | 0 | 0 | 0.0827 | 0.14 |
| 2 | 200 | 7 | 0 | 0 | 0.0827 | 0.14 |
| 3 | 200 | 21 | 0 | 0 | 0.0827 | 0.14 |
| 4 | 200 | 17 | 0 | 0 | 0.0827 | 0.14 |
| 5 | 200 | 25 | 0 | 0 | 0.0827 | 0.14 |
| 6 | 200 | 19 | 0 | 0 | 0.0827 | 0.14 |
| 7 | 200 | 16 | 0 | 0 | 0.0827 | 0.14 |
| 8 | 200 | 15 | 0 | 0 | 0.0827 | 0.14 |
| 9 | 200 | 11 | 0 | 0 | 0.0827 | 0.14 |
| 10 | 200 | 12 | 0 | 0 | 0.0827 | 0.14 |
| 11 | 200 | 22 | 0 | 0 | 0.0827 | 0.14 |
| 12 | 200 | 20 | 0 | 0 | 0.0827 | 0.14 |
| 13 | 200 | 17 | 0 | 0 | 0.0827 | 0.14 |
| 14 | 200 | 26 | 0 | 0 | 0.0827 | 0.14 |
| 15 | 200 | 18 | 0 | 0 | 0.0827 | 0.14 |
| 16 | 200 | 13 | 0 | 0 | 0.0827 | 0.14 |
| 17 | 200 | 15 | 0 | 0 | 0.0827 | 0.14 |
| 18 | 200 | 10 | 0 | 0 | 0.0827 | 0.14 |
| 19 | 200 | 14 | 0 | 0 | 0.0827 | 0.14 |
| 20 | 200 | 25 | 0 | 0 | 0.0827 | 0.14 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 21 | 200 | 19 | 0 | 0 | 0.0827 | 0.14 |
| 22 | 200 | 12 | 0 | 0 | 0.0827 | 0.14 |
| 23 | 200 | 6 | 0 | 0 | 0.0827 | 0.14 |
| 24 | 200 | 12 | 0 | 0 | 0.0827 | 0.14 |
| 25 | 200 | 18 | 0 | 0 | 0.0827 | 0.14 |
| 26 | 200 | 15 | 0 | 0 | 0.0827 | 0.14 |
| 27 | 200 | 20 | 0 | 0 | 0.0827 | 0.14 |
| 28 | 200 | 22 | 0 | 0 | 0.0827 | 0.14 |




The graph indicates that the process is in statistical control.

Ex: Twenty five boxes, each containing 20 electric switches were randomly selected and inspected for the number of defectives in each box. The number of defective found in each box were as follows:

| Box Number | No. of defectives | Fraction defectives |
|---|---|---|
| 1 | 3 | 0.15 |
| 2 | 2 | 0.10 |
| 3 | 1 | 0.05 |
| 4 | 0 | 0 |
| 5 | 4 | 0.20 |

| | | |
|---|---|---|
| 6 | 2 | 0.10 |
| 7 | 1 | 0.05 |
| 8 | 2 | 0.10 |
| 9 | 3 | 0.15 |
| 10 | 0 | 0 |
| 11 | 2 | 0.10 |
| 12 | 1 | 0.05 |
| 13 | 2 | 0.10 |
| 14 | 0 | 0 |
| 15 | 3 | 0.15 |
| 16 | 5 | 0.25 |
| 17 | 4 | 0.20 |
| 18 | 2 | 0.10 |
| 19 | 1 | 0.05 |
| 20 | 3 | 0.15 |
| 21 | 0 | 0 |
| 22 | 3 | 0.15 |
| 23 | 1 | 0.05 |
| 24 | 2 | 0.10 |
| 25 | 1 | 0.05 |

Calculate the control limits for np-chart.

Sol: here n=20, $\bar{p}$= 0.096, d= n$\bar{p}$= 20 x 0.096= 1.92

UCL= 1.92 + 3 $\sqrt{20 \text{ x } 0.096 \text{x } 0.904}$= 5.87

CL = 1.92

LCL= = 1.92 - 3 $\sqrt{20 \text{ x } 0.096 \text{x } 0.904}$= -2.03

**Ex:** Twenty two samples of bundles of a dozen of match boxes, were selected at regular intervals. The match boxes in each sample were inspected and the number of defects were noted down, with regard to the number of broken sticks, box shape, the labeling and the paste on the sticks etc. The number of defects were as follows:

| | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Sample No. : | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 |
| No. of defects: | 5 | 7 | 8 | 4 | 2 | 5 | 7 | 3 | 0 | 2 | 4 | 9 | 6 | 7 | 3 | 5 | 1 | 4 | 5 | 6 | 3 | 12 |

Prepare the control limits for C-chart.

Sol: $\sum c_i$=108,, k=22, $\bar{c}$= 108/22=4.91

UCL = 4.91 + 3 $\sqrt{4.91}$ = 11.56, CL = 4.91 and LCL = 4.91 - 3 $\sqrt{4.91}$= -1.74